CHARACTERIZATION OF NUMERICAL RELATIVITY WAVEFORMS OF ECCENTRIC BINARY BLACK HOLE MERGERS

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EXECUTIVE SUMMARY

An accurate description of the physics of eccentric binary black holes (BBHs) throughout the late-inspiral, merger, and ringdown requires numerical relativity (NR). Once NR simulations are post-processed and extracted [1], it is necessary to quantify the eccentricity and other orbital parameters that uniquely identify them. Existing methods to measure eccentricity based on the BH trajectories in NR simulations do not provide a sound approach, given that these trajectories are gauge-dependent. The research team circumvents this limitation by introducing a gauge-invariant method that characterizes an NR waveform by comparing it to a large array of semianalytical waveforms, dubbed ENIGMA [2] waveforms, that are written in the gauge used to detect gravitational waves with the Laser Interferometer Gravitational-Wave Observatory detectors. The researchers quantify the circularization of eccentric BBHs near merger and quantify the impact of higher-order waveform modes in the morphology of eccentric NR waveforms. This study is timely and relevant to characterizing future observations of eccentric BBHs.

RESEARCH CHALLENGE

This research aims to quantify the eccentricity and mean anomaly of NR waveforms that describe eccentric BBH mergers using a gauge-invariant approach and to develop open source algorithms to conduct this analysis on Blue Waters to process catalogs of NR waveforms at scale. The work is of interest to the NR and gravitational wave (GW) astrophysics communities.

METHODS & CODES

The research team explored a variety of gauge-invariant objects to directly compare NR and ENIGMA (eccentric, nonspinning, inspiral, Gaussian-process merger approximant) waveforms. The researchers found that the dimensionless object $M\omega$, where ω is the mean orbital frequency and M stands for the total mass of the BBH, provides a robust approach to capture the signatures of eccentricity. To compute $M\omega$ for NR waveforms, the team uses the relation $M\omega = \frac{1}{2}\frac{\dot{a}\,b - a\,\dot{b}}{a^2+b^2}$, where a and b represent the plus and cross polarizations of an NR waveform h=a-ı b. The ENIGMA waveform model produces this gauge-invariant quantity by providing, as input parameters, the mass-ratio of the BBH system and the initial eccentricity, e_0, and mean anomaly, l_0, of the system at a fiducial GW frequency f_0 from which the waveform is produced.

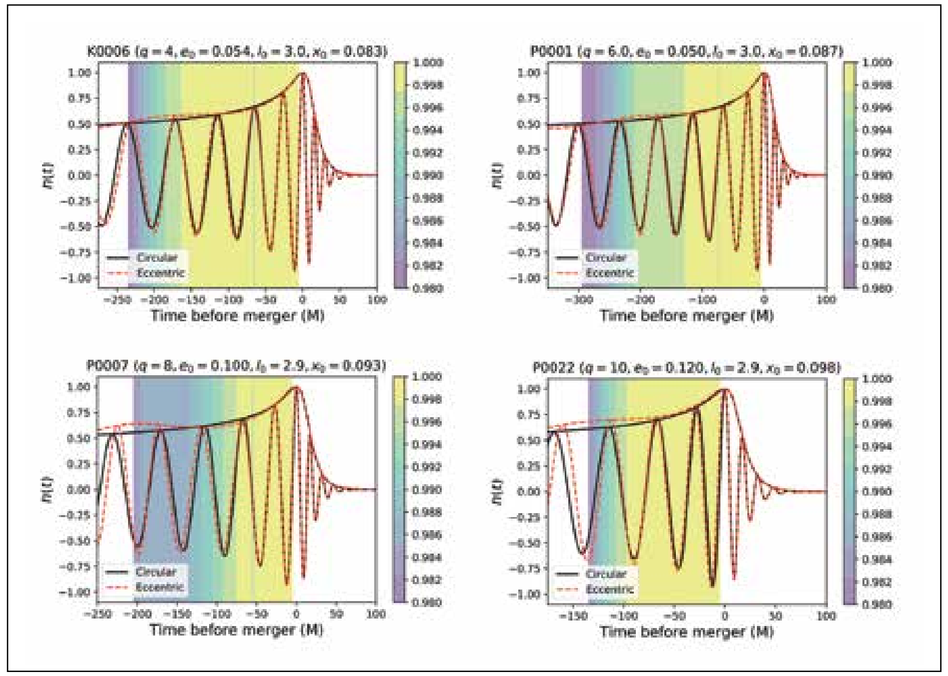

Figure 1: Inner product, indicated by the color bar, between eccentric binary black hole systems and their quasicircular counterparts. As the eccentric systems reach merger, marked at t=0M, eccentricity is gradually radiated away until the systems become effectively circular, which corresponds to an overlap value of 1 with quasicircular waveforms.

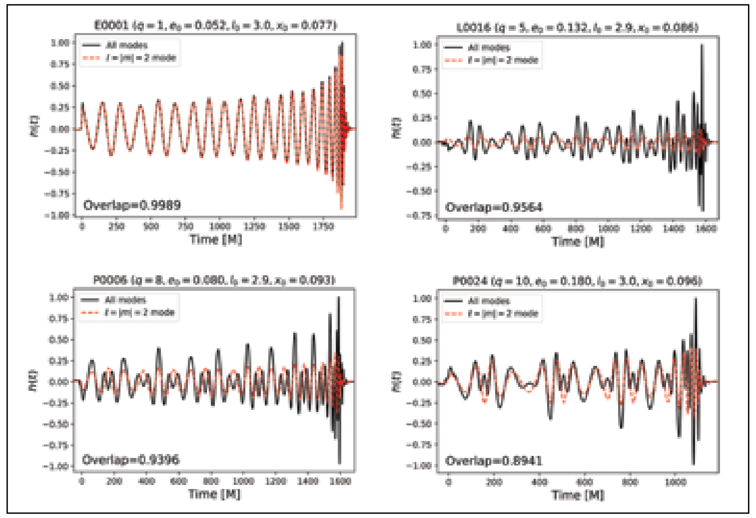

Figure 2: Overlap between numerical relativity waveforms that include either the modes (l, |m|) = {(2, 2), (2, 1), (3, 3), (3, 2), (3, 1), (4, 4), (4, 3), (4, 2), (4, 1)}, or just the l=|m|=2 mode. Higher-order modes become more significant for asymmetric mass-ratio binary black holes.

The team removes junk radiation from the NR waveform by applying a Savitsky–Golay filter. Since input parameters are required to produce ENIGMA waveforms, seed values are initialized for f_0 and e_0. The researchers provide an informed guess of the GW frequency using the relation $f_0 \sim \omega_0/(\pi M)$, where $\omega(t=t_0)=\omega_0$, and t_0 is the time at which the NR waveform is free from junk radiation. Mean anomaly is initialized to π, a value manually determined to be optimal through verification of a few NR waveforms. Orbital eccentricity does not require a seed value since the range of possible values is consistent for all catalogued waveforms.

The algorithm starts with a grid search in the 2D parameter space of (f_0, e_0), and iteratively refines it. The researchers densely sample the frequency range $f \in [f_0\text{-}5\text{Hz}, f_0\text{+}5\text{Hz}]$ and the eccentricity range $e \in [0.1, 0.3]$. For each coordinate pair, an ENIGMA $M\omega$ is produced using the specified (f_0, e_0) values and the seeded l_0. The resulting $M\omega$ time evolution is then compared to that of the original NR waveform. Parameters are chosen that minimize the difference between two properties of the ENIGMA and NR evolutions: time duration of the first orbital cycle and the maximum change in $M\omega$ during the first cycle. Throughout the entire search, l_0 is held constant.

After completing the grid search, the chosen (f_0, e_0) parameters are further refined iteratively. In this stage, initial GW frequency and orbital eccentricity are independently varied stepwise to increase precision. The team used this method to characterize the 89 NR waveforms presented in [3].

RESULTS & IMPACT

This research addresses two topics: (1) the rate of circularization of eccentric BBH mergers, and (2) the importance of including higher-order waveform modes for an accurate modeling of these astrophysical systems.

Regarding the first analysis, Fig. 1 indicates that for the more eccentric systems, circularization only happens about 50M before merger, and that the increase in overlap, as $t_0 \rightarrow 0M$, is not monotonic. Rather, it has an oscillatory behavior that tracks the eccentric trajectory of the BBH system, which is clearly captured by the waveform amplitude. As soon as the waveform amplitude of the eccentric signals becomes increasingly monotonic, so does the overlap.

For the second topic, the researchers have constructed NR waveforms that include either the modes (l, |m|) = {(2, 2), (2, 1), (3, 3), (3, 2), (3, 1), (4, 4), (4, 3), (4, 2), (4, 1)}, or just the l=|m|=2 mode. For the NR waveforms that include higher-order modes, the team has quantified the regions of parameter space that maximize the contribution of these modes for GW detection. Upon constructing these NR waveforms, the scientists compute the overlap between these NR waveforms so that those only include the leading-order quadrupole term. Fig. 2 presents results for these calculations for a variety of astrophysically motivated scenarios. These indicate that the inclusion of higher-order modes does not quantitatively modify the morphology of l=|m|=2 NR waveforms that describe equal-mass eccentric BBH mergers. However, NR waveforms that describe asymmetric mass-ratio and eccentric BBH mergers have a much richer topology that requires the inclusion of higher-order waveform modes.

WHY BLUE WATERS

Blue Waters was critical in producing a catalog of NR waveforms and enabling the research team to postprocess and characterize these waveforms *in situ* and at scale. While this analysis would be prohibitive in a campus cluster-sized resource, the analysis can be completed within a few minutes on Blue Waters for a catalog of over 100 NR waveforms.

PUBLICATIONS & DATA SETS

S. Habib and E. A. Huerta, "Characterization of numerical relativity waveforms of eccentric binary black hole mergers," *Phys. Rev. D*, vol. 100, no. 4, pp. 044016–044026, Aug. 2019.