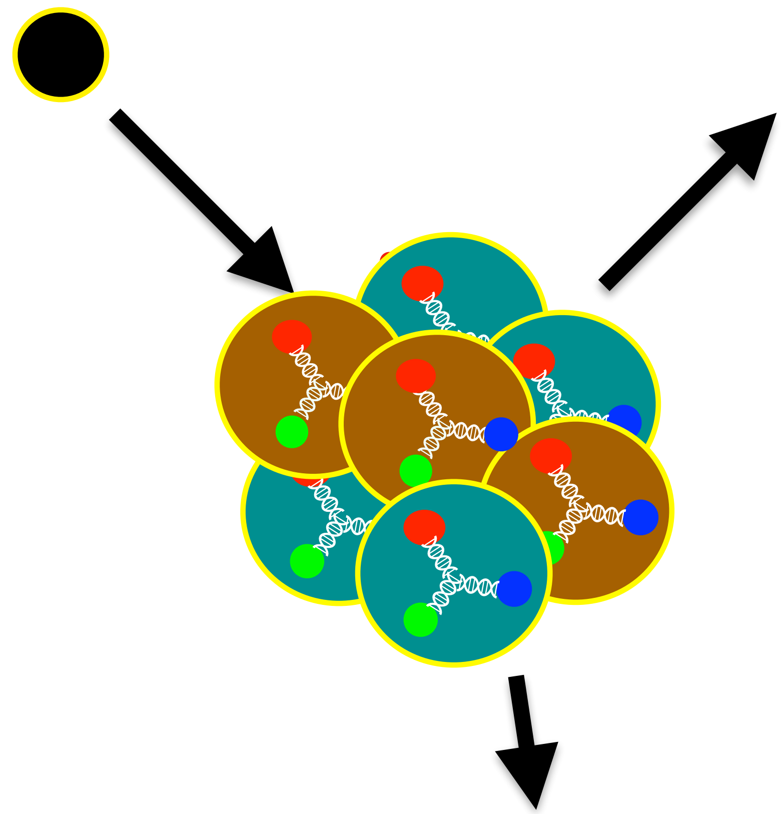
Dark Matter Interactions with Nuclei

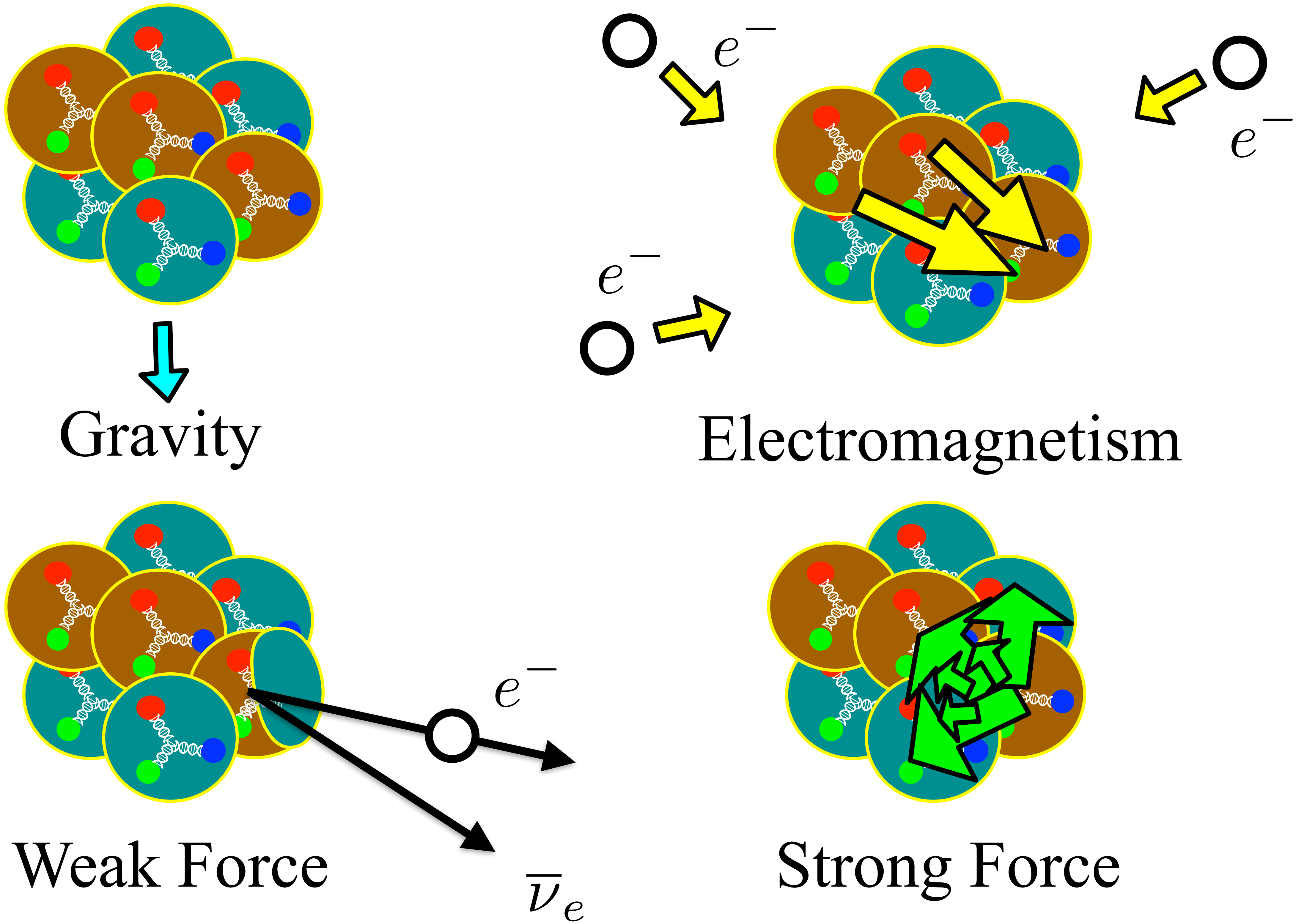


Michael Wagman

P.I. Phiala Shanahan

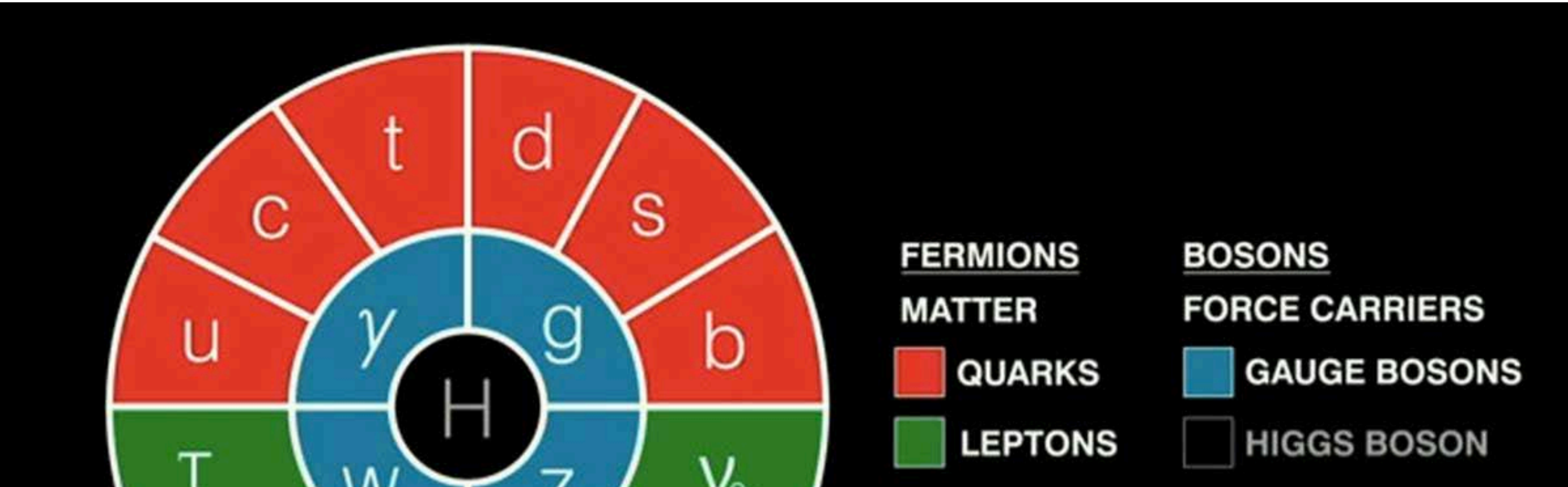
Blue Waters Symposium 2019

What we know



Gravity
e^-
e^-
e^-
Electromagnetism
e^-
$\overline{\nu}_e$
Weak Force
Strong Force

What we know



t
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ν_μ
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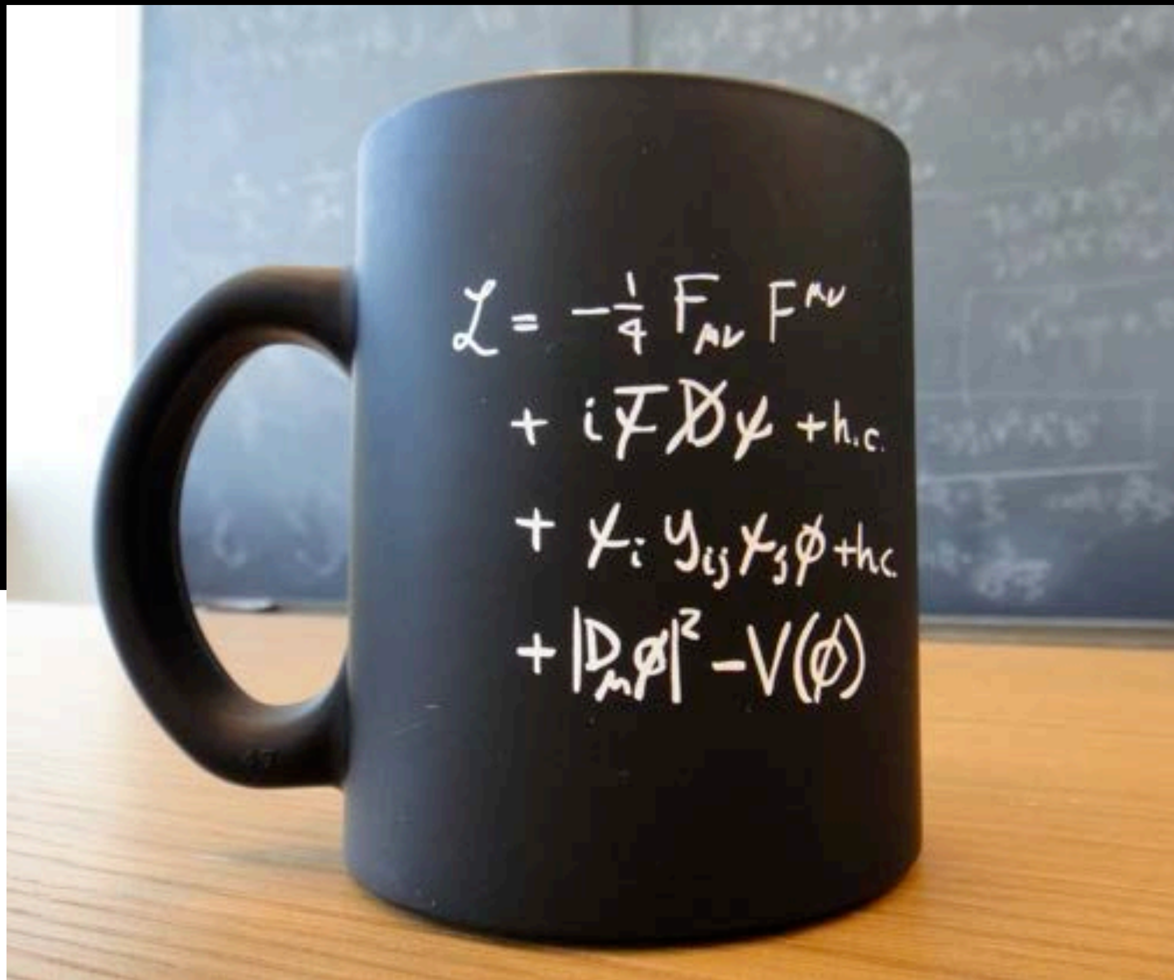
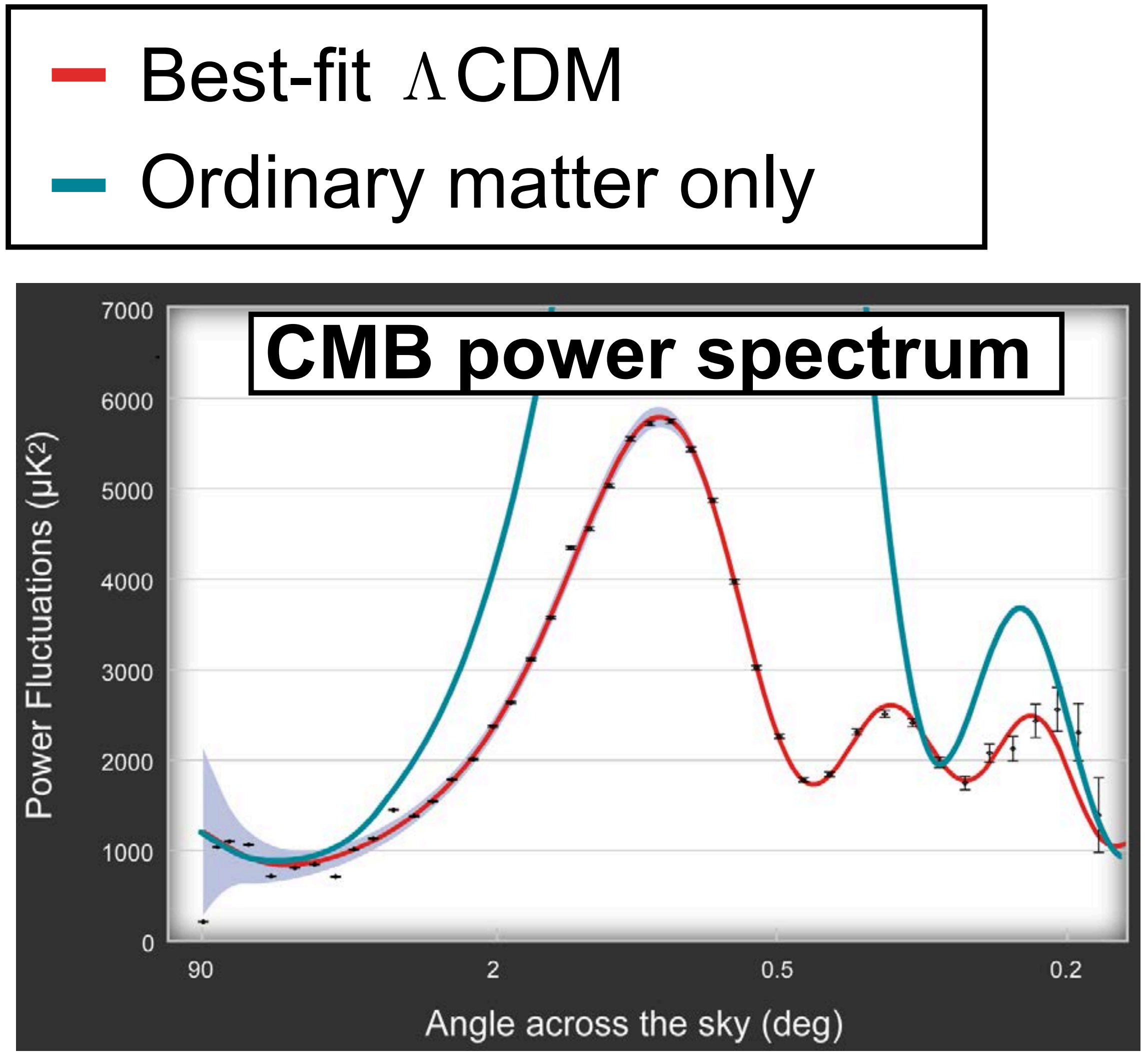
FERMIONS
MATTER
QUARKS
LEPTONS
BOSONS
FORCE CARRIERS
GAUGE BOSONS
HIGGS BOSON



$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i\bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

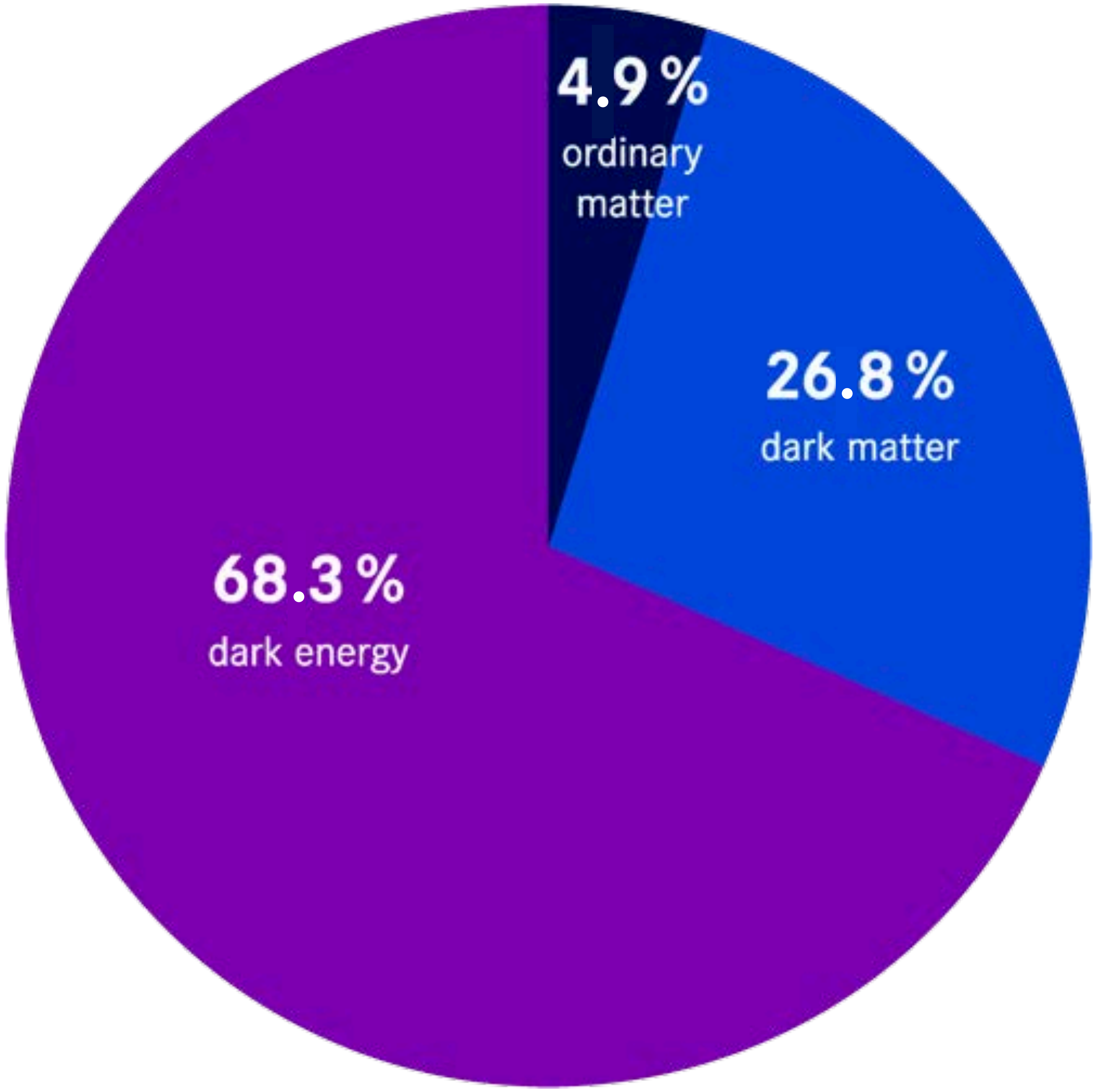
What we don't know

Astrophysical and cosmological observations show that most of the universe is not ordinary Standard Model matter



Black Points - WMAP data

https://lambda.gsfc.nasa.gov/education/cmb_plotter/



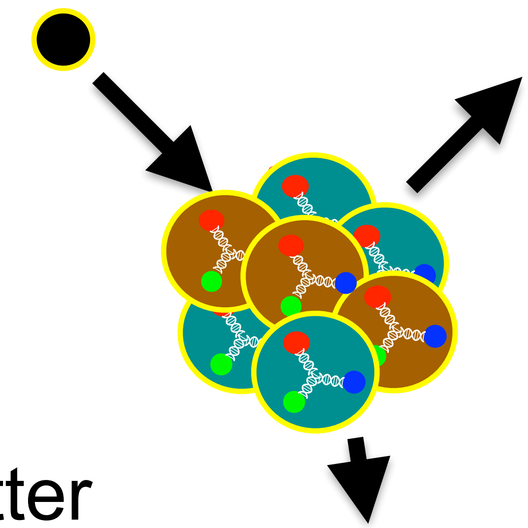
What we don't know

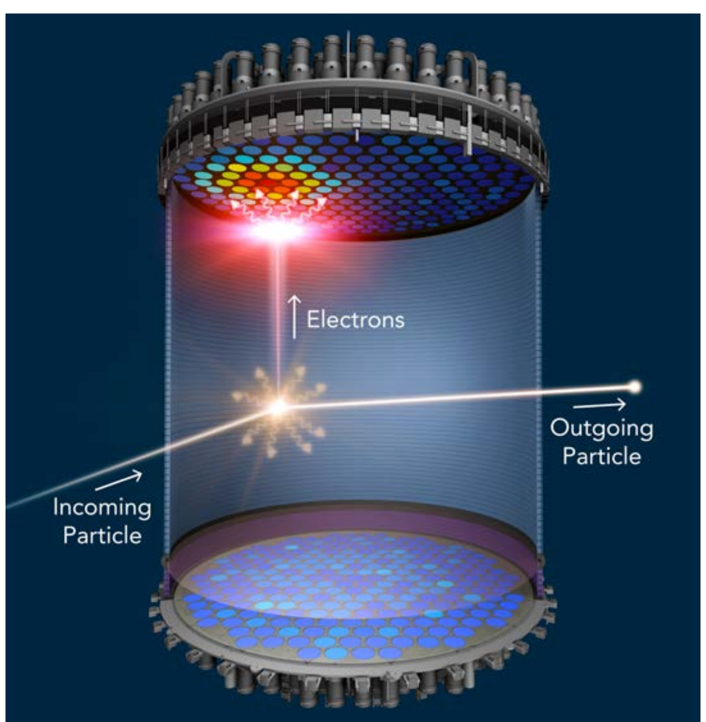
Dark matter direct detection

Experiments look for nuclei recoiling from scattering with something invisible

Heavy nuclei are often used to maximize sensitivity

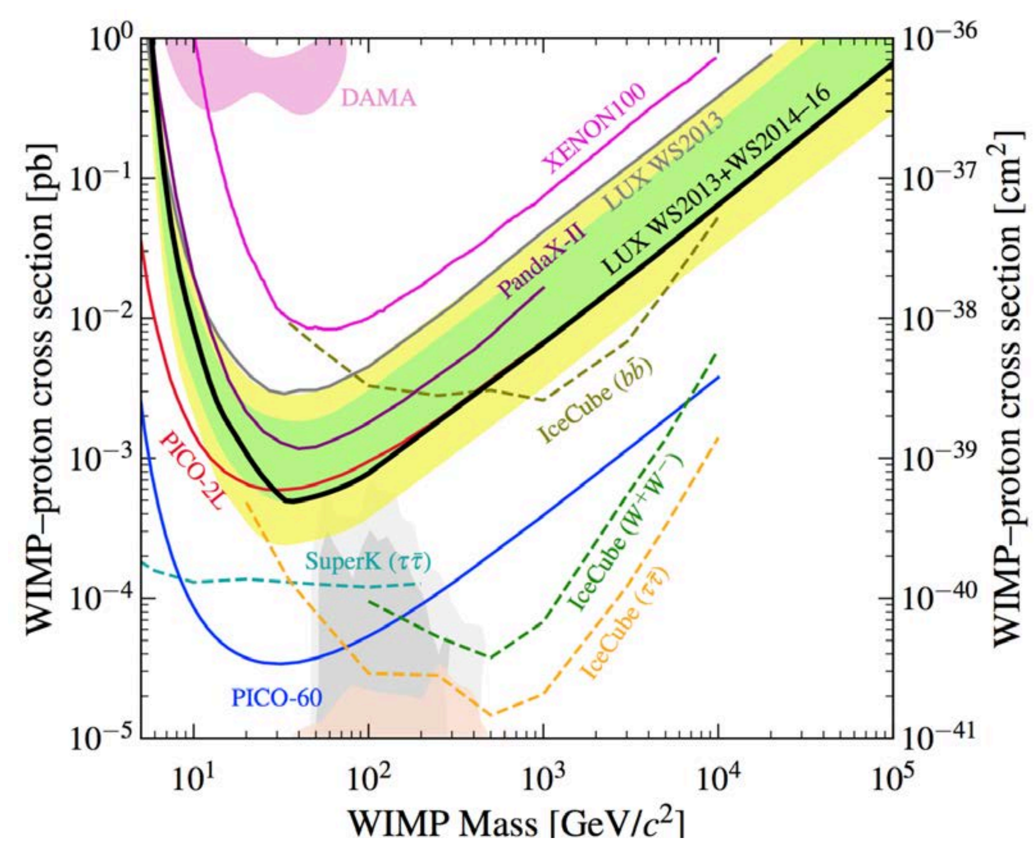
Standard Model theory needed to relate nucleus - dark matter interactions with proton (or quark) - dark matter interactions





LUX

Theory assumptions

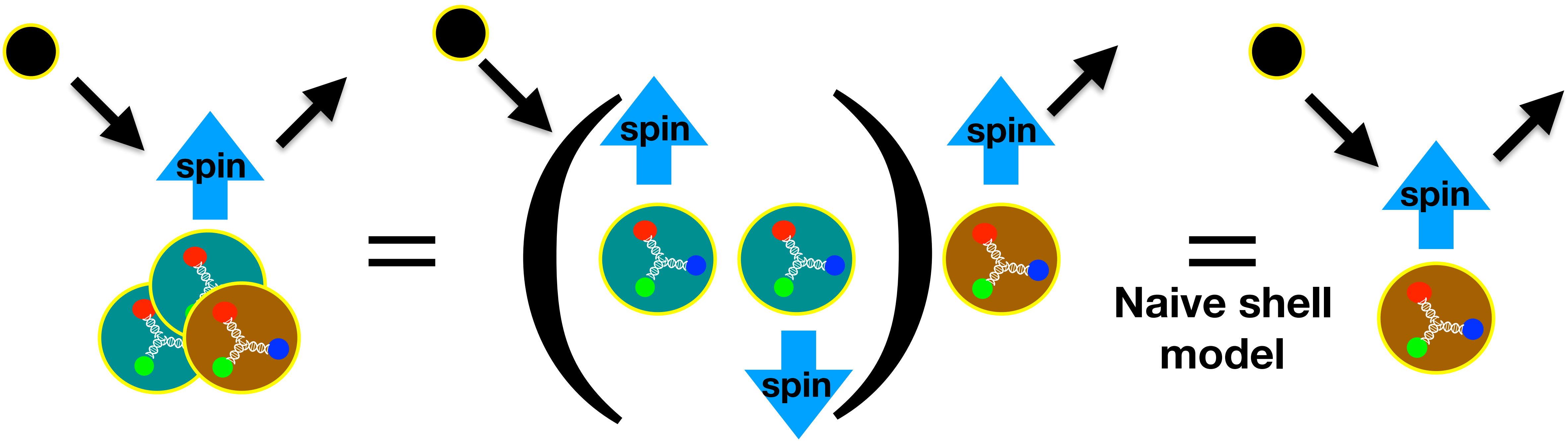


Akerib et al (LUX), PRL 118 (2017)

Naive nuclei

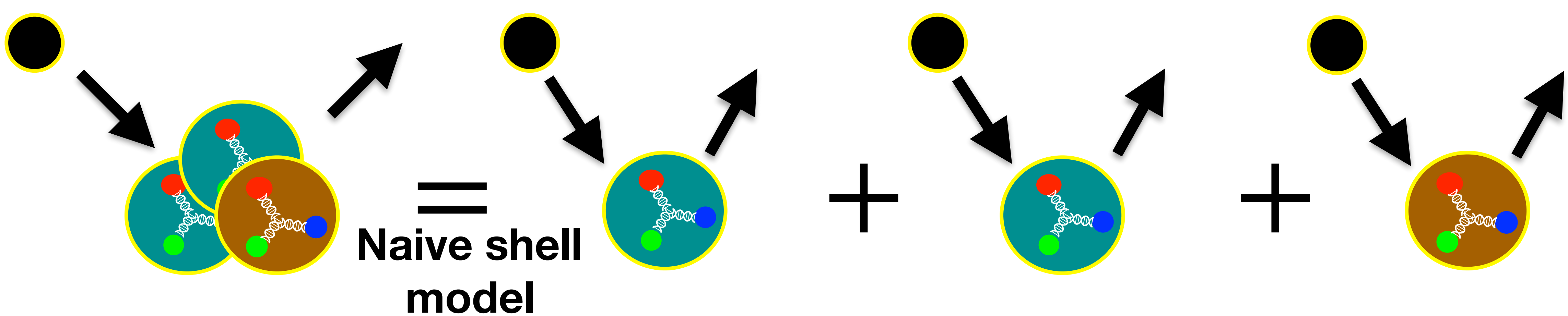
Nuclei = naive shell model + QCD effects

Axial / **tensor** currents couple to nuclear spin



Scalar currents couple to total quark number of nucleus,

— Dominate spin-independent dark matter scattering



Solving the Standard Model

Quantum observables can be represented by path integrals

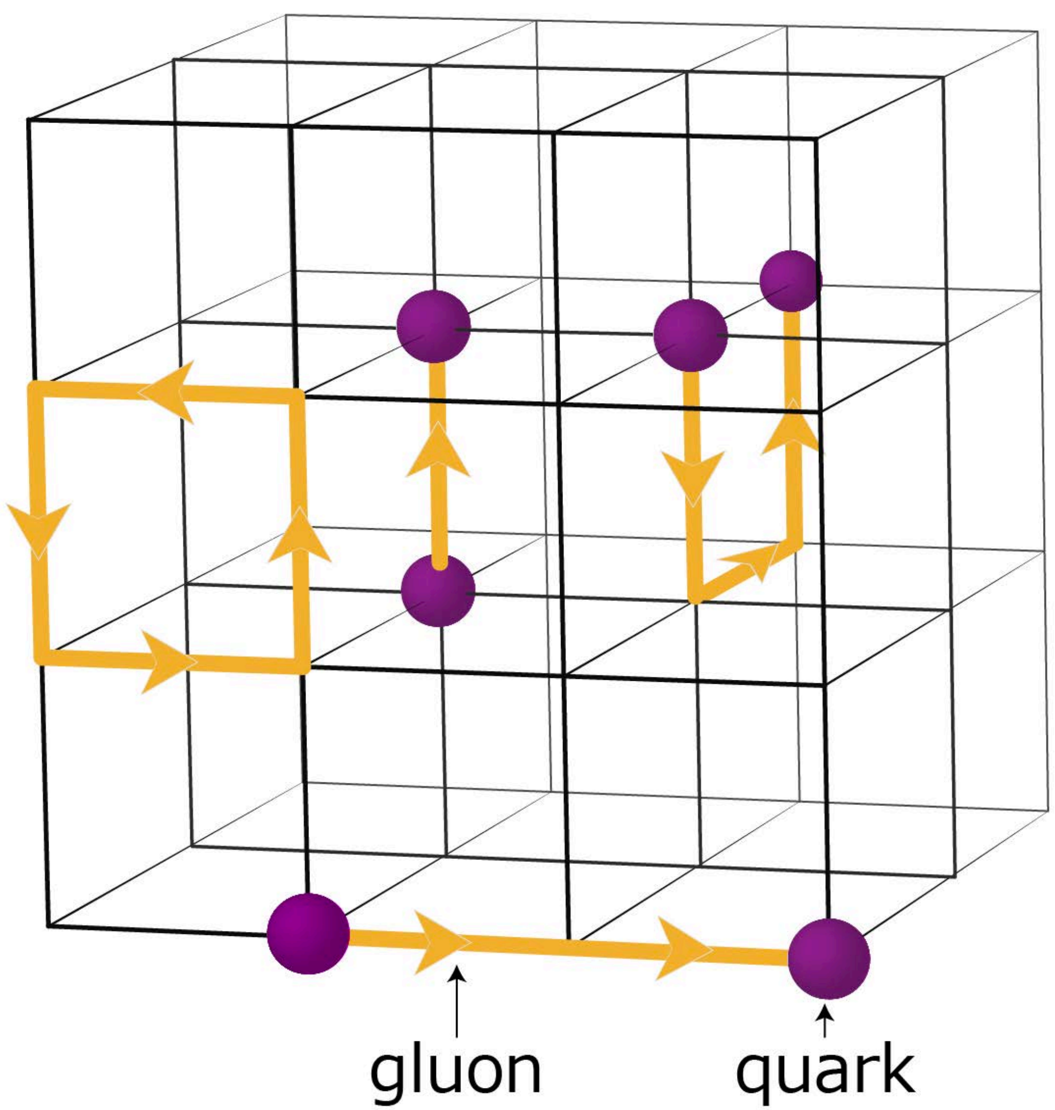
$$< \mathcal{O} >= \int_{\text{paths}} d(\text{path})\ \mathcal{O}(\text{path})$$

If continuous spacetime is replaced with a finite-size discrete lattice of points, path integrals become well-defined multidimensional integrals

Quark path integral in QCD analytically calculable

$$Z = \int \mathcal{D}U \mathcal{D}\overline{q} \mathcal{D}q\ e^{-S_{QCD}(U,q,\overline{q})}$$

$$= \int \mathcal{D}U\ e^{-S_G(U)} \det(\not{D}(U) + m_q)$$



Gluon path integral performed numerically with Monte Carlo sampling

Quark propagators

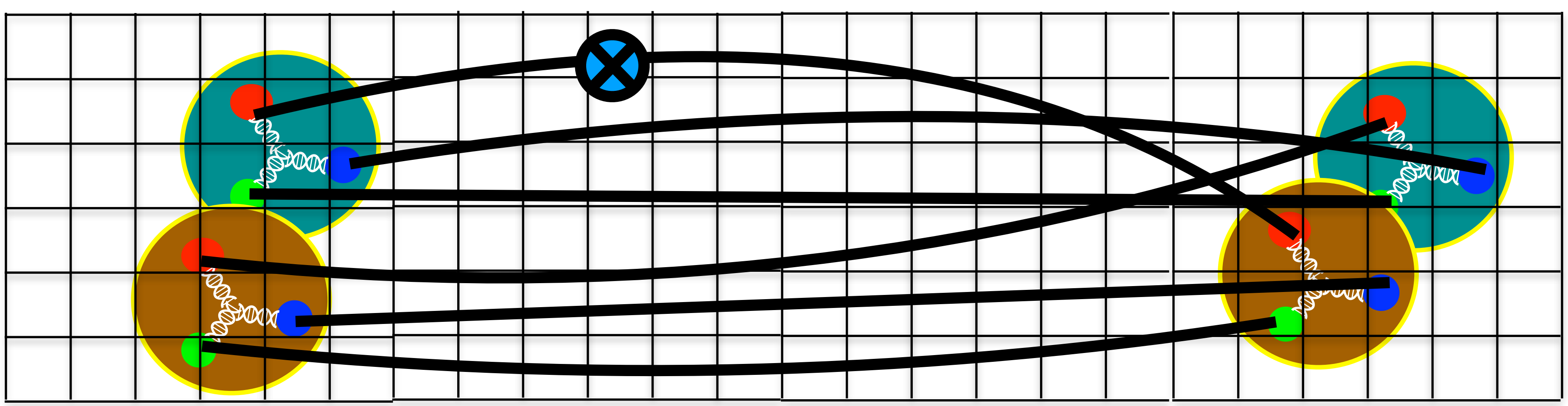
For a fixed gluon field configuration, quark propagators calculable with linear algebra

$$< q(x)\overline{q}(y) >= (\not{D} + m_q)^{-1}_{xy}$$

Usually more efficient to solve linear equations with a given source, "point-to-all" propagator

$$(\not{D} + m_q)_{xy} q(y) = s(x)$$

Repeatedly solving this linear system for a $10^9 \times 10^9$ sparse matrix often dominant computational cost



More complex observables built from tensor products quark propagators

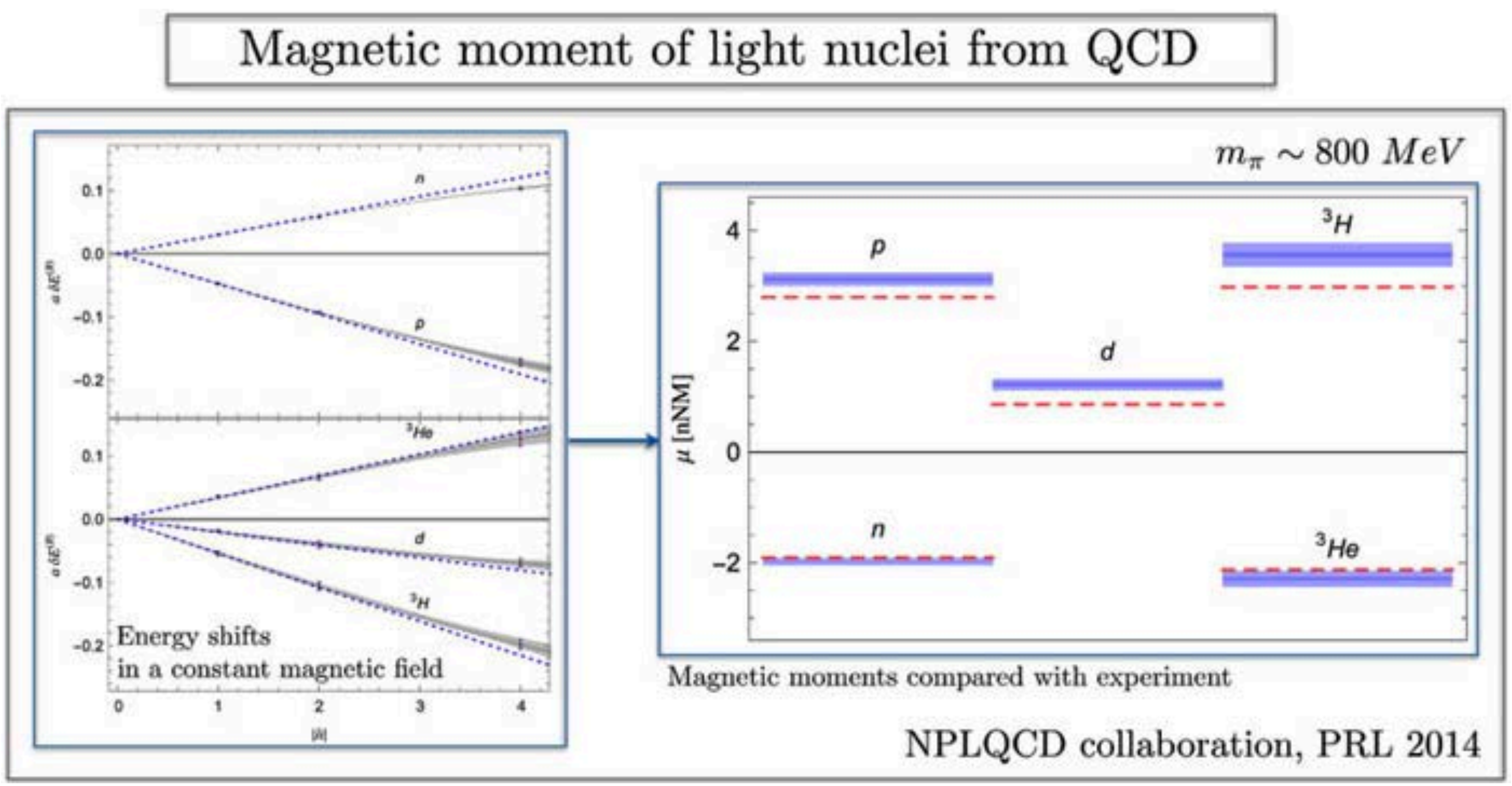
From quarks to nuclei

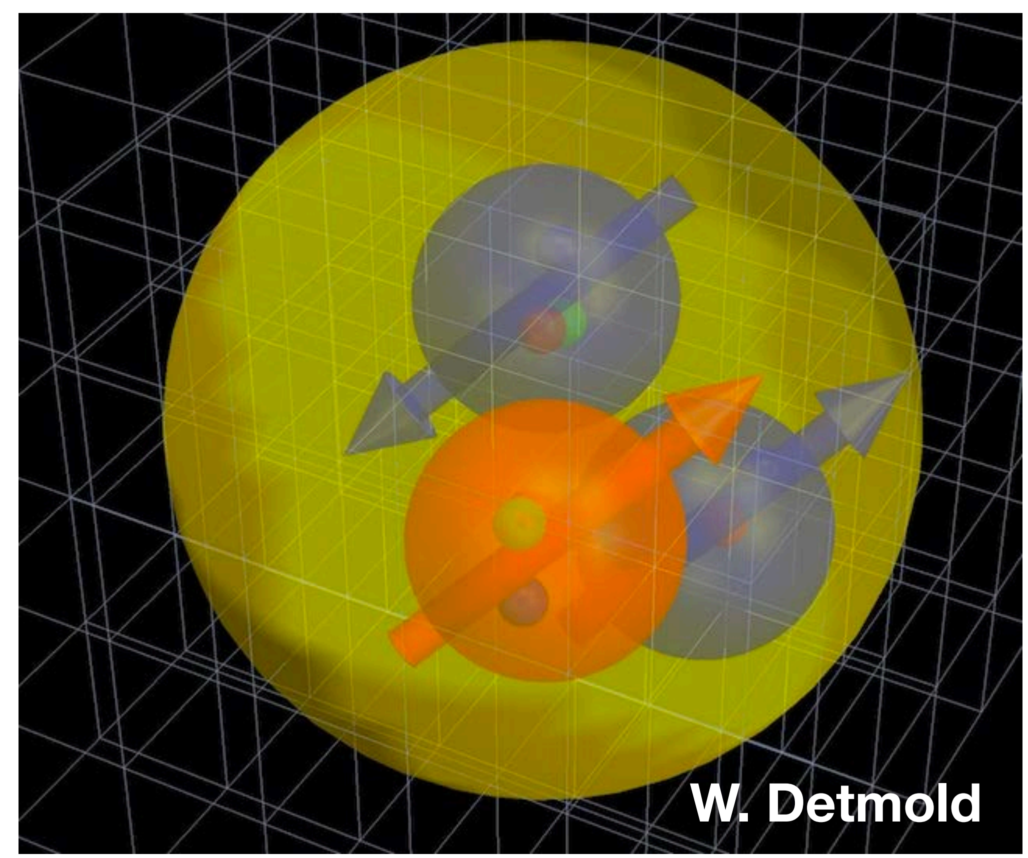
Nuclei can be constructed from lattice QCD path integrals by contracting quark propagators with appropriately symmetrized nuclear wavefunctions

NPLQCD, PRL 87 (2013) | Yamazaki et al, PRD 86 (2012) | Detmold and Orginos, PRD 87 (2013) | Doi and Endres, Comput. Phys. Commun. 184 (2013) …

Calculations performed at unphysically heavy quark masses so far

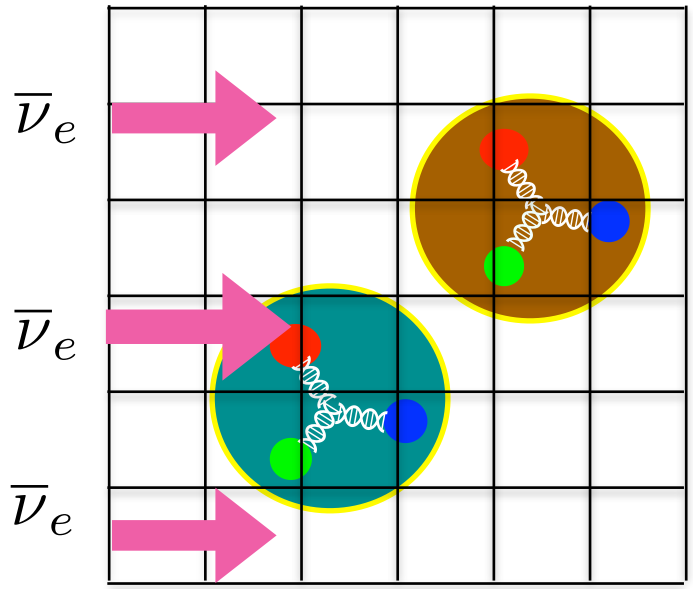
Naive shell model describes the magnetic moments of light nuclei in nature and at heavier quark masses with surprisingly good accuracy



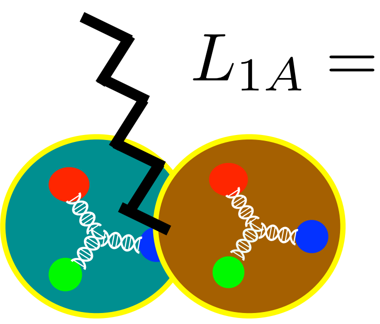


Nuclear interactions

Coupling of nucleus to external probe (photon, W-boson, …) obtained from response to background field



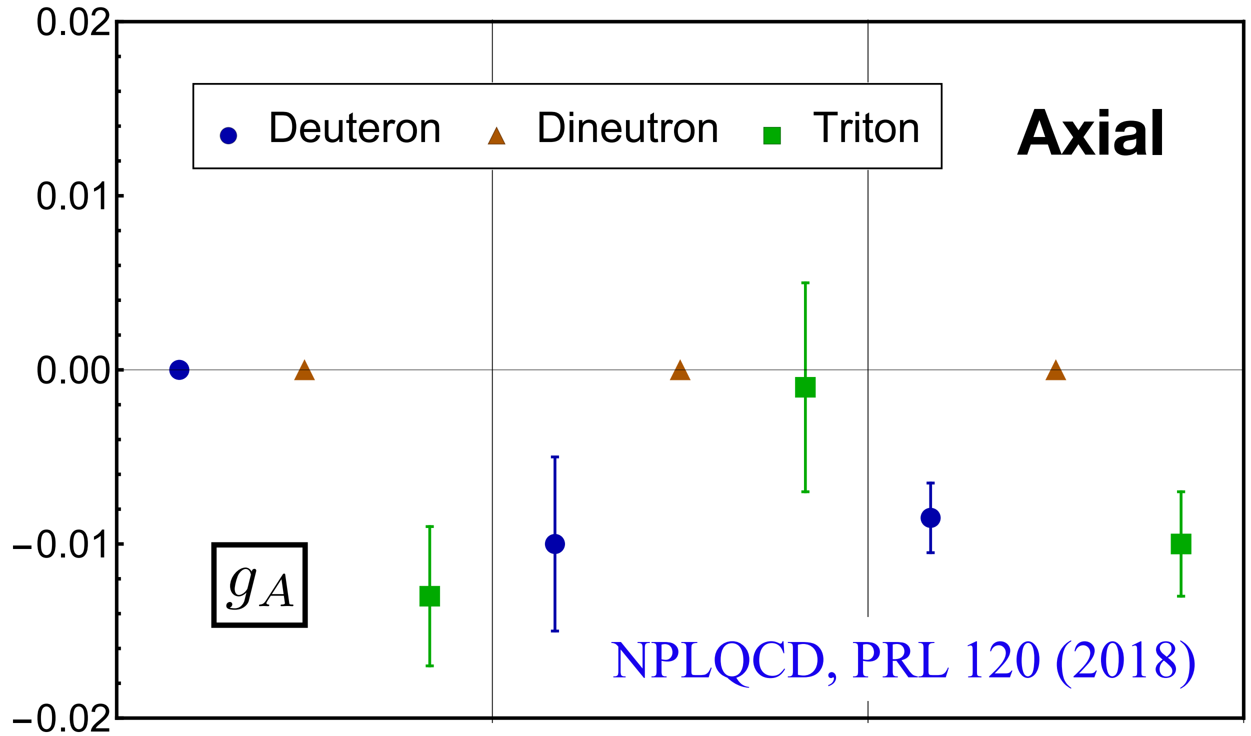
Useful for computing experimentally inaccessible observables, e.g. proton-proton fusion rate



$$L_{1A} = 3.9(0.2)(1.0)(0.4)(0.9)\ \ \mathrm{fm}^3$$

NPLQCD, PRL 119 (2017)

$$\frac{\Delta g_A^{(u-d)}}{4S_3T_3g_A^{(u-d)}} \quad \frac{\Delta g_A^{(u+d+s)}}{2S_3g_A^{(u+d+s)}} \quad \frac{\Delta g_A^{(u+d-2s)}}{2S_3g_A^{(u+d-2s)}}$$



W-boson (axial) couplings of light nuclei differ from naive shell model productions by O(1%)

$m_\pi \sim 800$ MeV

Dark matter and nuclei

QCD effects reduce scalar couplings to light quarks by 1(1)% with nucleon number A=2 and 4(1)% with A=3

Scalar coupling to strange quarks reduced by 10(4)% in ^{3}H

— Dominant coupling in some BSM models

NPLQCD, PRL 120 (2018)

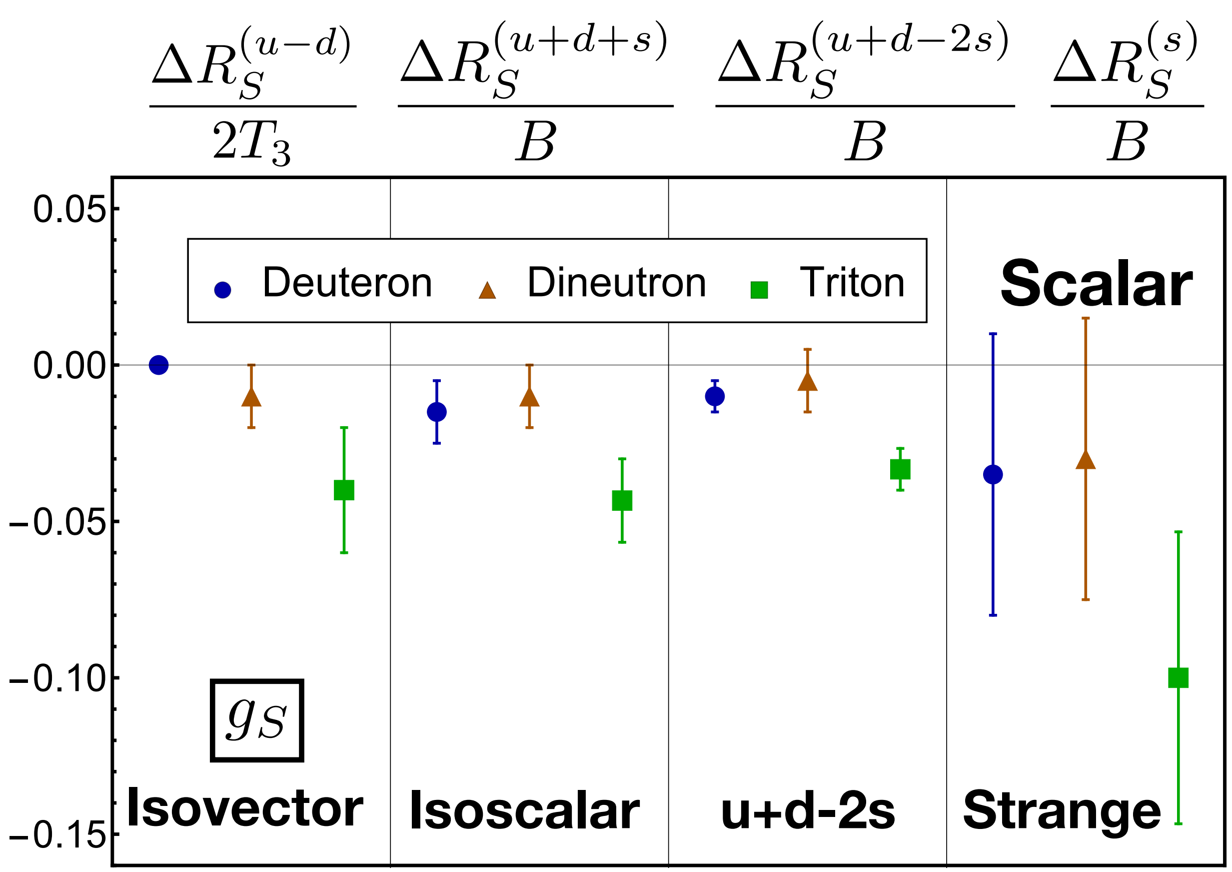
Are QCD effects on scalar couplings generically larger than axial couplings?

If yes, QCD effects important for interpreting dark matter direct detection experiments

Physical quark mass QCD results will test / inform models of experimentally relevant large nuclei

Hoferichter, Klos, Menéndez, Schwenk, PRD 94 (2016)

Fieguth et al, PRD 97 (2018)



$m_\pi \sim 800$ MeV

Challenges ahead

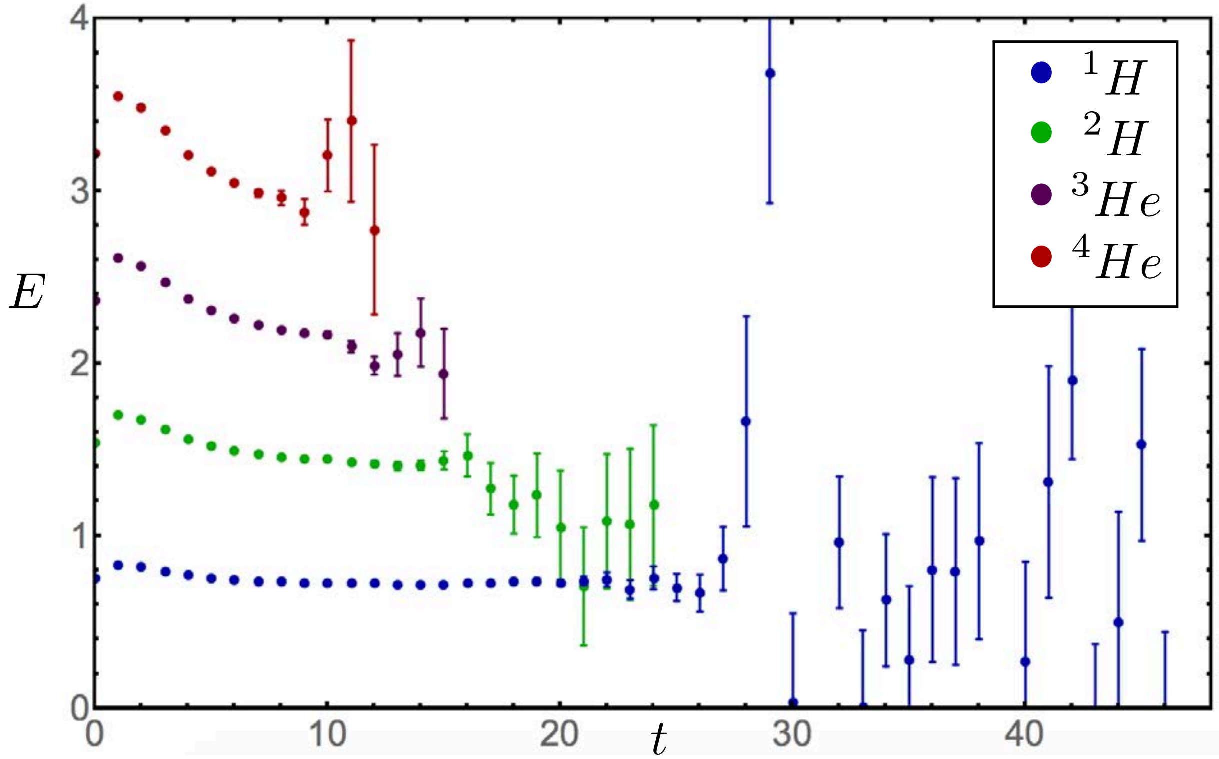
Simple algorithms for light quark propagators inefficient at lighter quark masses

— multigrid algorithms

Monte Carlo noise grows exponentially as quark masses are reduced

$$\frac{\text{signal}}{\text{noise}} \propto e^{-A\left(M_N - \frac{3}{2} m_\pi\right) t}$$

— high-statistics studies targeting important observables

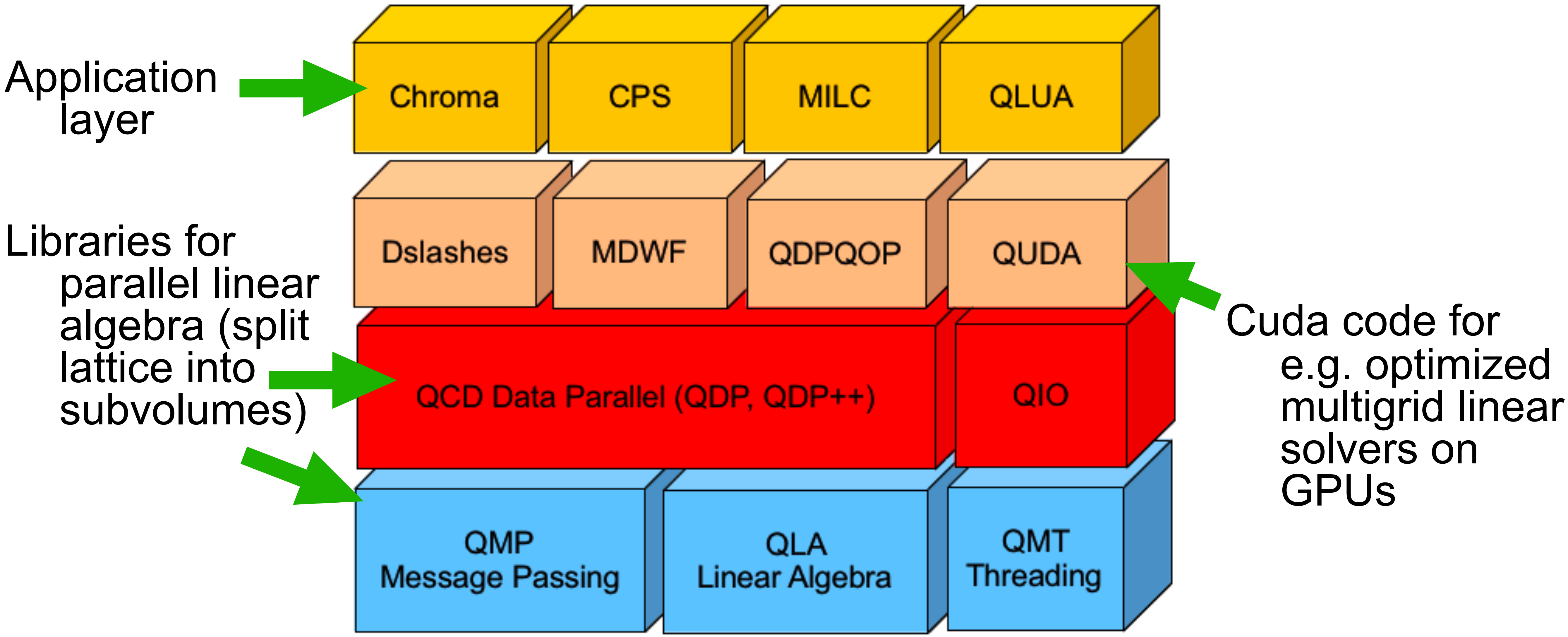


Large lattice volumes require large amounts of memory

Most computational steps are “just” linear algebra, GPUs efficient

— ideally suited for parallel computation on many Blue Waters GPU nodes

Software for solving QCD



QDP-JIT uses JIT compilation with LLVM or PTX to port QDP to GPUs

— effectively ports Chroma to GPUs and avoids Amdahl's law issues

Winter, Clark, Edwards, Joó, arXiv:1408.5925

Gluon field generation

Problem 1: gluon field configuration space is big

10^9 dimensional for $64^3 \times 128$ lattice

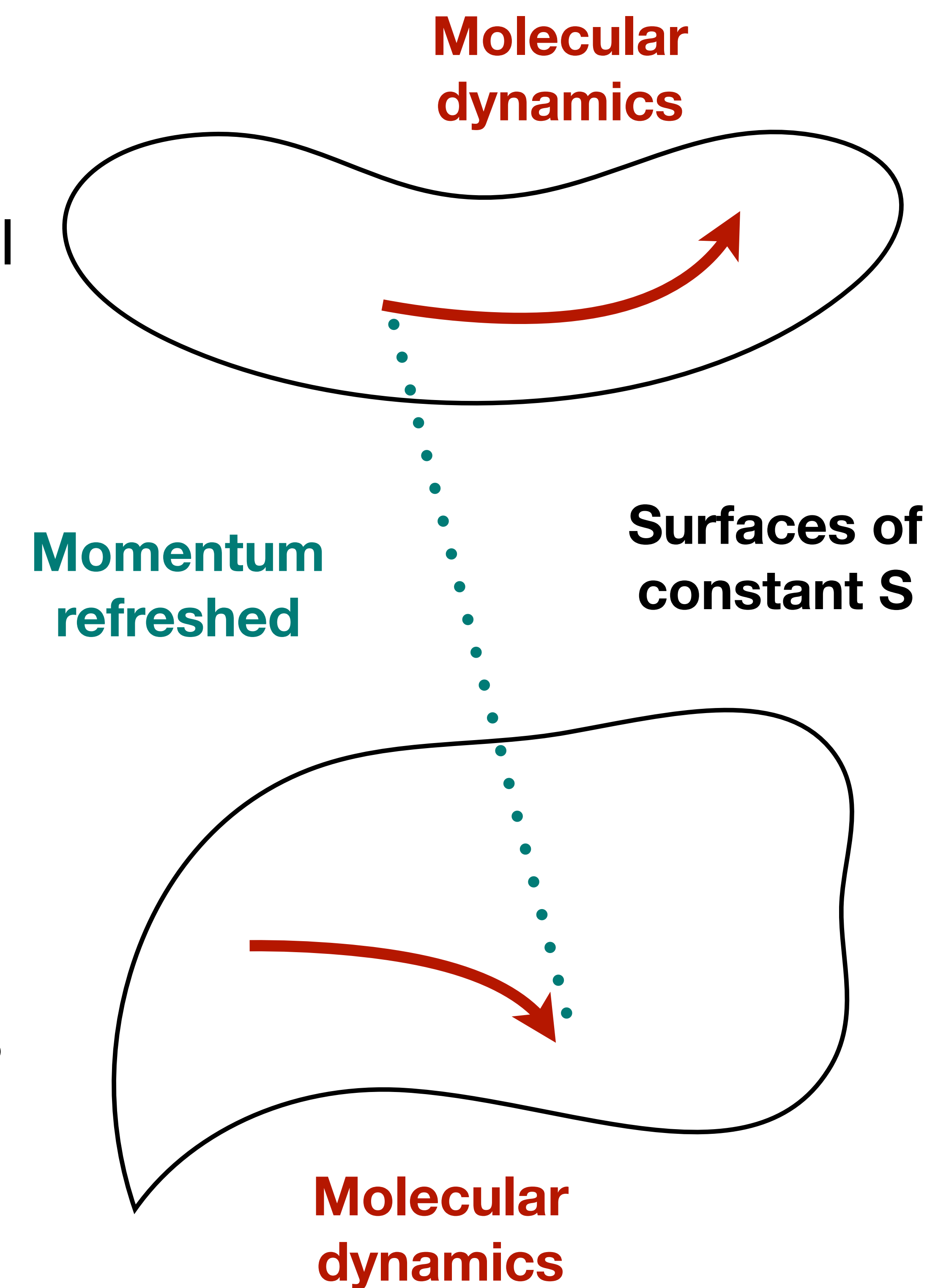
Problem 2: gluon effective action is non-local

$$\text{Prob}(U) = e^{-S_G(U) + \ln\det(\not{D}(U) + m_q)}$$

Solution: **hybrid Monte Carlo**

molecular dynamics with energy -> action

1) Refresh momenta to random values
2) Evolve fields along surface of constant S with classical molecular dynamics
3) Accept/reject new field configuration



Dealing with determinants

Calculating full determinant of Dirac operator impractical

Pseudofermions: $\det(\not{D} + m_l)^2 = \int \mathcal{D}\varphi^\dagger \mathcal{D}\varphi e^{-\varphi^\dagger (\not{D}+m_l)^{-1} \varphi}$

HMC performed in gluon and pseudofermion configuration space

Light quarks — linear solves become expensive

Hasenbusch preconditioning: $\det(\not{D} + m_l) = \frac{\det(\not{D} + m_l)}{\det(\not{D} + m_0)} \det(\not{D} + m_0)$

Multigrid!

Hasenbusch, Phys. Lett. B 519 (2001)

Strange quarks — pseudofermions only work for squared determinants

Rational HMC: use an approximate square root $\det(\not{D} + m_s) \approx \sqrt{\det(\not{D} + m_s)^2}$

$$\sqrt{x} \approx \alpha_0 + \sum_i \frac{\alpha_i}{x + \beta_i}$$

Clark and Kennedy, Nucl. Phys. Proc. Suppl. 129 (2004)

Contributions from all poles efficiently calculated using multi-shift solvers

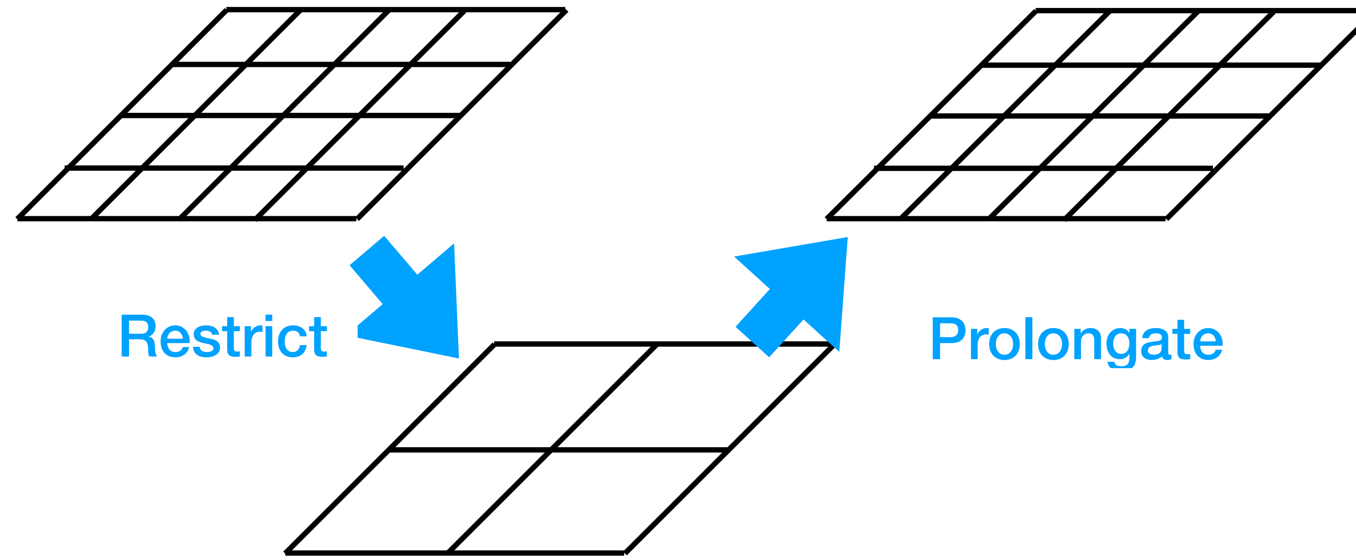
Frommer et al, Int. J. Mod. Phys. C 6 (1995)

Multigrid

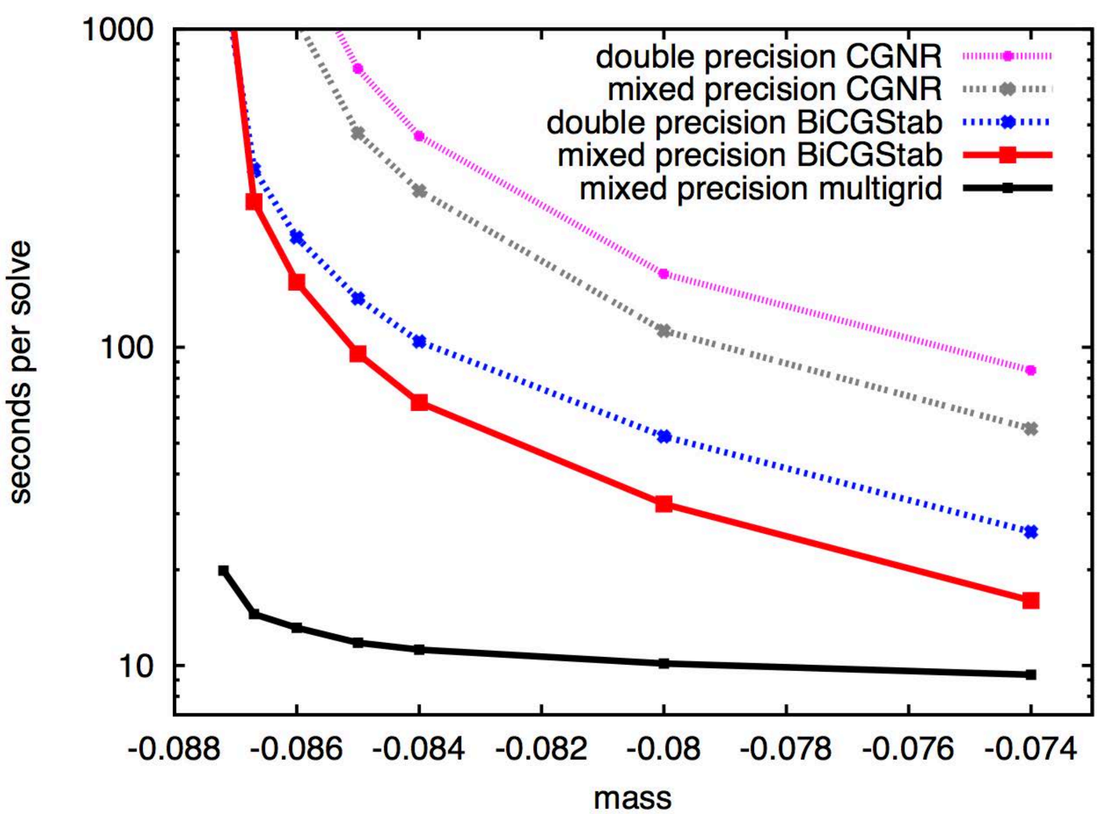
Low modes of the Dirac operator are responsible for “critical slowing down”

Low modes less low on coarser grid

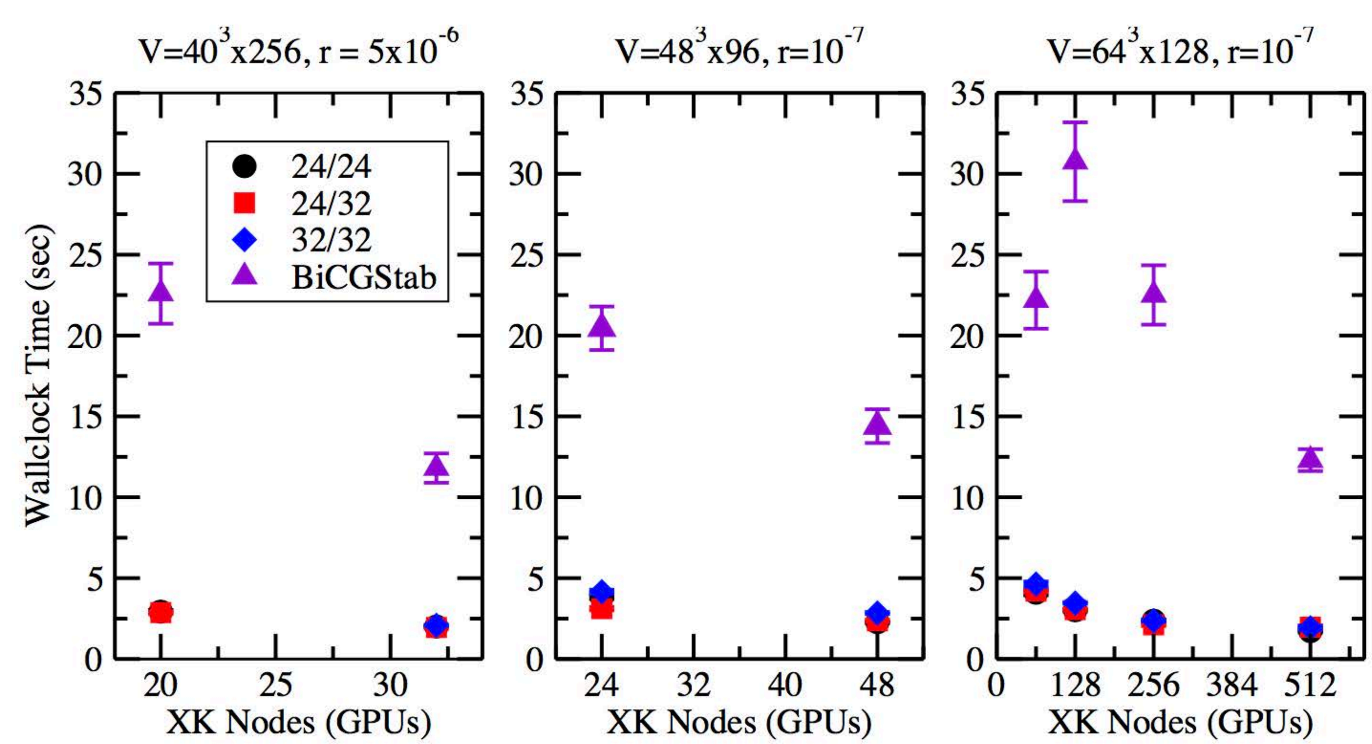
Coarse-grid solution smoothed and prolongated, provides initial guess (preconditioner) for fine-grid solve



Babich et al, PRL 105 (2010)



Clark et al, SC 16 Article 68 (2016)



Practical speedups of 10x or more for light quark solvers

HMC performance on BW

$48^3 \times 96$ lattice, **192 nodes**

Light quark solver:	822 s
Strange quark solver:	167 s
HMC force assembly:	492 s
Total:	1481 s

$64^3 \times 128$ lattice, **512 nodes**

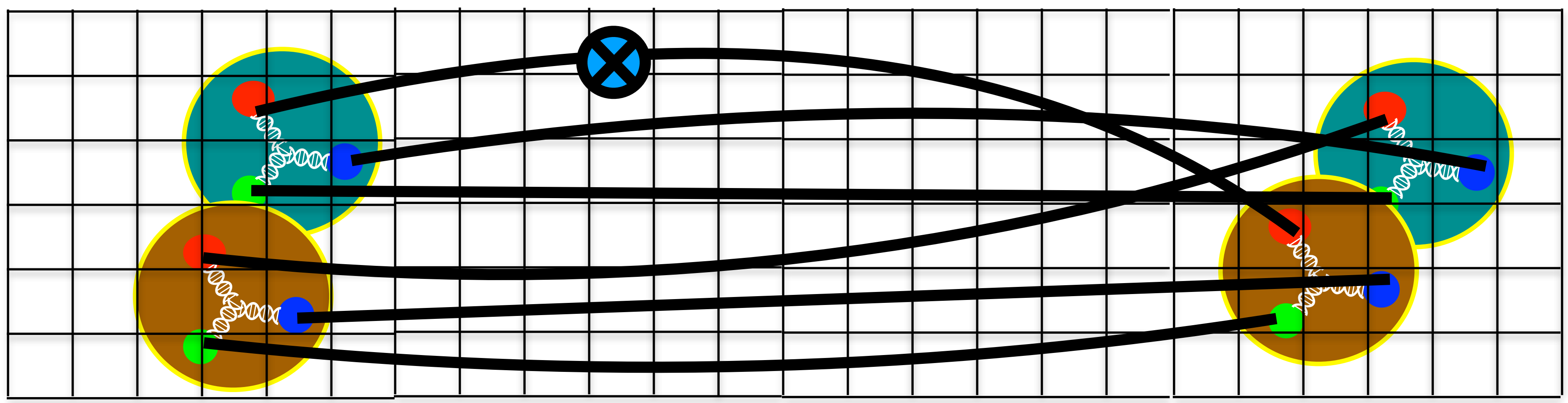
Light quark solver:	418 s
Strange quark solver:	182 s
HMC force assembly:	568 s
Total:	1168 s

Large memory needs for multigrid and QDP-JIT → many node jobs

Light quark solver pieces: ($48^3 \times 96$)

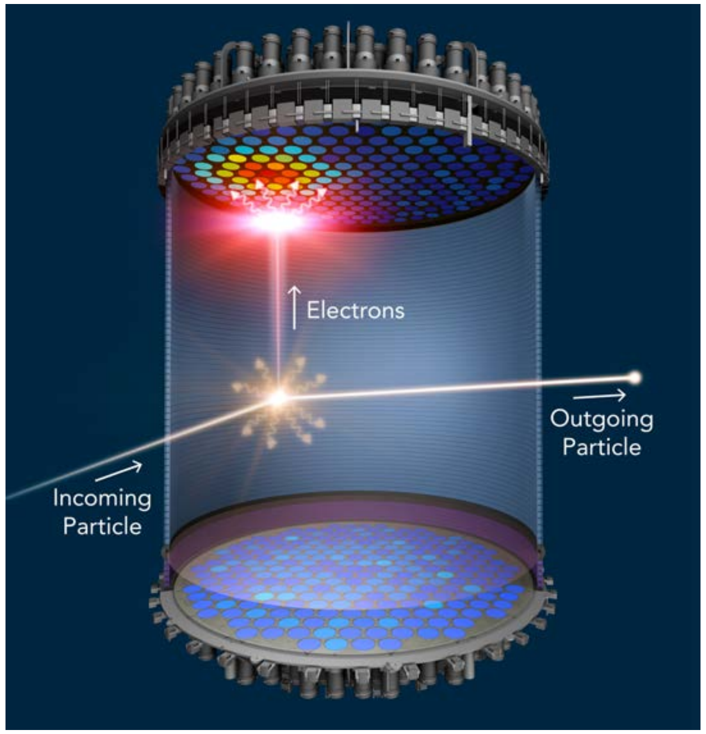
25%	multigrid null space
(7+8+11+18+25)%	Hasenbusch ratios
7%	other QUDA operations

Building nuclei from quarks on BW



One quark propagator per gluon field		**512 quark propagators per gluon field**	
$48^3 \times 96$ lattice, **128 nodes**		$48^3 \times 96$ lattice, **128 nodes**	
Quark wave functions:	14 s	Quark wave functions:	7,168 s
Multigrid nullspace:	**579 s**	Multigrid nullspace:	**579 s**
Light quark solver:	6 s	Light quark solver:	3,072 s
Baryon blocks:	33 s	Sparse baryon blocks:	169 s
Total:	**632 s**	Total:	**10,988 s**

Impact



Our Blue Waters running began April, 2019

— *results will provide insight into QCD effects on scalar currents needed to reliably interpret dark matter direct detection experiments*

Same lattices useful for other calculations (fusion rates, double beta-decay, …)

— *lattices will be made publicly available to broaden impact of Blue Waters production*

Calculations of nuclei with physical quark masses possible thanks to efficient algorithms and 100+ node Blue Waters GPU jobs