Improved Initial Lapse and Shift for Binary Black Hole Simulations

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Why Blue Waters?

Over the next seven months, I will use Blue Waters to perform full Numerical Relativity simulations of binary black hole mergers in order to study the effects of a new set of initial data for two equations used during the calculation of gravitational waveforms.

- Full NR simulations require supercomputers to evolve 10 coupled nonlinear partial differential equations on a 3D grid - computationally very expensive.
- Necessary to have high resolution around each black hole.
- Blue Waters can handle large-scale, complex simulations.
- Knowledgeable support system, both about the system and the software used for evolutions.
- Many simulations we will compare to have already been done on BW.

Talk Outline

- Introduction
- Key challenges and research goals
- Why it matters
- Analytic trumpet initial data
- Preliminary results
- Analytic spin correction terms
- Expected results
- Conclusions and future work

Introduction

To perform these NR simulations, we use the Einstein ToolKit framework.

- We evolve the BSSN 3+1 equations of General Relativity, with choice for gauge:

$$(\partial_t - \beta^i \partial_i)\alpha = f(\alpha)K = -2\alpha K$$
$$\partial_t \beta^i = \frac{3}{4}\tilde{\Gamma}^i - \eta\beta^i$$

  with $\alpha =$ lapse, β^i=shift, and K=extrinsic curvature.
- These are first order differential equations, and therefore require specification of initial data.

Introduction

In general, for initial data, we use

$$\alpha_0 \approx \Psi_4 \Psi_2 \text{ and } \beta_0^i = 0$$

These initial data can also be freely chosen, although different choices may lead to slightly different evolved values of the lapse and shift.

- We would like to choose initial data that mimics the settled shape of lapse and shift using the initial data above.
- This will hopefully allow the gauge to settle to its final shape more quickly.

Key Challenges

We would like to exploit the fact that we can choose initial data freely to construct fully analytic initial values for the lapse and shift that improve the accuracy and speed of challenging simulations.

To do this:

- First, study and model the late-time behavior of the lapse and shift, then construct equations that mimic these behaviors to use as initial data.
  - These new initial data will be referred to as "trumpet" or "trumpet + spin".
- Then, apply the initial data to an over-resolved case ($q = 1/3$ mass ratio, nonspinning).
  - No expectation of reduction of spurious radiation in the waveforms.
  - Expectation of a reduction in error without increasing resolution.
- Now: apply initial data to a spinning case.

Research Goal

The nonspinning $q = 1/3$ case gave the expected results, which will be shown.

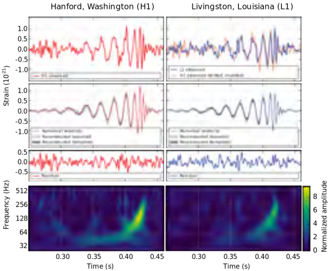
- We are now moving on to study a spinning case with equal masses ($q = 1$) and moderate spin ($a = 0.8$), using the Blue Waters system.

Our ultimate goal is to apply this to more challenging cases.

- HISpID data with spins of $a \approx 0.99$
- Small mass ratios $1/100 < q < 1/10$.
- High energy collisions $p/M = 0.99$.

Why it Matters / Broader Impact

In 2015, the Laser-Interferometer Gravitational-wave Observatory detected the first BBH merger[1].



Current predictions expect detections to occur with a frequency of up to a few per week[2].

[1] Abbott et. al. *Observation of Gravitational Waves from a Binary Black Hole Merger (2016).*

[2] https://dcc.ligo.org/public/0150/G1800370/005/O3_rates_amsterdam.pdf

Why it Matters / Broader Impact

More challenging areas (high spin, small mass ratio) are sparsely covered, we now need to fill more of the parameter space.

- These simulations can be done, but take weeks to months of supercomputer time; increasing resolution slows them down substantially.
  - Want to gain accuracy without increasing grid resolution.
  - Do not want to increase computational resources needed.

These results will allow evolution of difficult simulations without an increase in computational expense, helping to fill the parameter space in time for LIGO's predicted detections.

Methods

To evaluate the performance of these new initial data choices ("trumpet" and "trumpet + spin"), we need to perform full Numerical Relativity simulations of the system through merger[3]. Results will be quantified in several ways:

- Deviations from zero of the L2-norm of the Hamiltonian and Momentum constraint violations.
- Reduction in spurious (junk) radiation in waveforms, and speed of convergence to extrapolated waveform.
- Speedup and weak/strong scaling for each method.

For each system, we need three resolutions to calculate convergence.

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[3] We will first test on well-studied cases before moving on to more challenging simulations.

So how do we calculate these new initial data?

We look at the evolved shape of the lapse and shift, and construct Pade approximants to mimic their behavior, in the hope of speeding up the settling of the gauge.

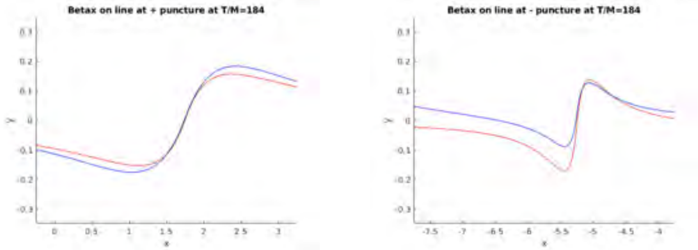


Figure: Shift on a line through the large BH (left) and small BH (right) for a nonspinning system with $q = 1/3$. Red line is an approximant to the lapse, blue is the evolved lapse.

Analytic forms of α_0 and β_0^i for nonspinning black holes

The analytic forms of the initial lapse and shift are

$$\alpha_0 = \frac{a}{1 + b\psi^n + c\psi^{n+1} + d\psi^{n-1}}$$

$$\beta_0^{r_i} = \frac{A(\psi_i - 1)^2}{1 + B\psi_i + C\psi_i^2 + D\psi_i^3}$$

$$\beta_0^r = \sum_{i=1}^{N} \beta_0^{r_i}$$

where N is the number of black holes, and the conformal factors are defined by $\psi = 1 + \sum_{i=1}^{N} \frac{m_i}{2r_i}$ and $\psi_i = 1 + \frac{m_i}{2r_i}$.

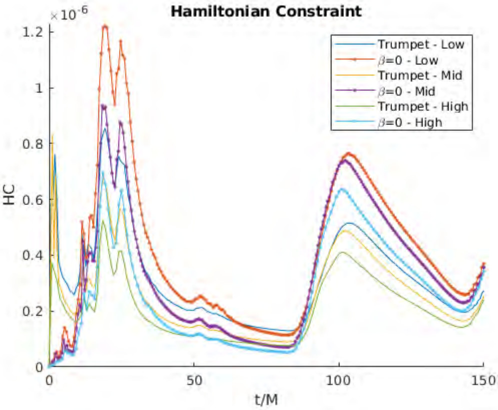
Analytic forms of α_0 and β_0^i

The parameters A, B, C, D and a, b, c, d are matching parameters found by matching the expressions on the previous slide in the following way:

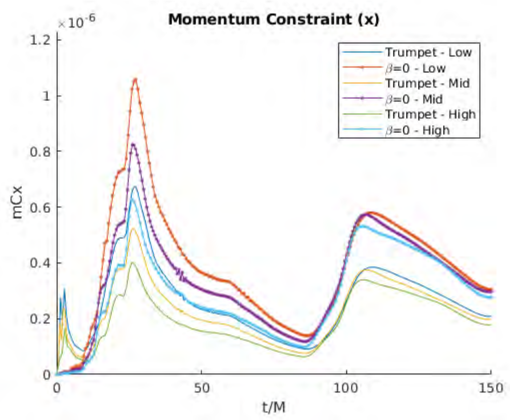
- Asymptotically, to the 1+log slice of the Schwarzschild (nonrotating) lapse and shift.
- As $r \to 0$, to trumpet-sliced lapse and shift.

The initial shift is then rotated into Cartesian coordinates. So how does this method work on the overresolved $q = 1/3$, $a = 0$ case?

Preliminary Results



Preliminary Results



Spin Correction Terms

To construct the spin correction terms, we need to consider the conformal Kerr metric in Cartesian coordinates.

$$g_{\mu\nu} = \begin{bmatrix} \frac{\sigma-1r^2}{\rho_z^2} & \frac{a_z\sigma_z y}{\rho_z^2} & -\frac{a_z\sigma_z x}{\rho_z^2} & 0 \\ \frac{a_z\sigma_z y}{\rho_z^2} & 1+a_z^2 h_z y^2 & -a_z^2 h_z xy & 0 \\ -\frac{a_z\sigma_z x}{\rho_z^2} & -a_z^2 h_z xy & 1+a_z^2 h_z x^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

We can invert (1) and use the fact that

$$g^{\mu\nu} \begin{bmatrix} -\frac{1}{\alpha^2} & \frac{\beta^i}{\alpha^2} \\ \frac{\beta^j}{\alpha^2} & \gamma^{ij} - \frac{\beta^i\beta^j}{\alpha^2} \end{bmatrix}$$

to get the spin correction terms for lapse and shift[4].

[4] These require rotations so that they are valid for arbitrary spin orientation, not just spins along the z−axis.

Expected Results Using Spin Correction Terms

- We expect to see reductions in the peaks of the L2-norms of the Hamiltonian and momentum constraint violations
- We expect reductions in junk radiation in the early part of the gravitational waveform
- Faster convergence to the extrapolated (to ∞) waveform.

Approximate Runtime of Current Spinning Simulations on BW

We are currently running three simulations on BW, all with $q = 1$, $a = 0.8$, for convergence studies.

Coarsest Grid Spacing	Nodes	Runtime (hours)	Runtime (node hours)
5.45..	8	408	3265
4	8	588	4705
3.33..	16	667	10666

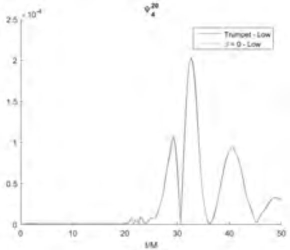
All use the trumpet + spin initial gauge. Columns 3 and 4 are the expected runtimes in wallclock hours and node hours (resp.) over the life of the run, projected from the current simulation speed per hour.

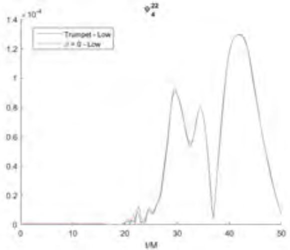
Conclusions and Future Work

- The preliminary case $q = 1/3$, nonspinning, with trumpet initial data reduces error without increasing grid resolution.
- We expect to see similar results using the trumpet+spin initial data on a moderately spinning $a = 0.8$, $q = 1$ system.
  - Runs of this type are currently in progress on BW.
  - Early results show small reduction in spurious radiation in the gravitational waveforms.
- This method will be applied to more challenging cases, which we will be able to begin to test, once we are confident in our results, using the BW allocation.

Extra Slides

$\Psi_4^{2,0}$ and $\Psi_4^{2,2}$ for $q = 1/3$, $a = 0$ Run





Early Constraint Violations for $q = 1$, $a = 0.8$ Run

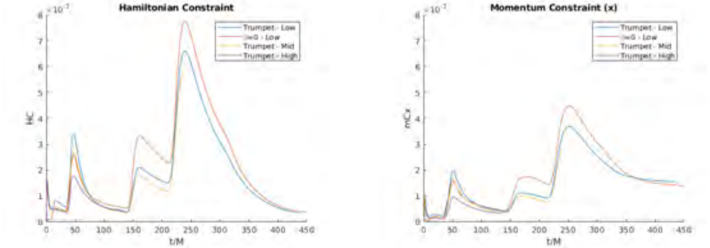
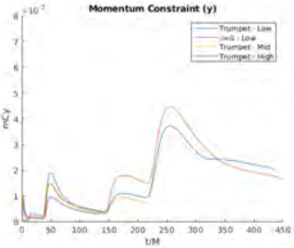


Figure: Deviations from zero of the L2-norm of the Hamiltonian constraint violation and the x-component of the momentum constraint violation. Dotted lines are simulations using the original $\beta = 0$ initial data, and solid lines use trumpet initial data.

Early Constraint Violations for $q = 1$, $a = 0.8$ Run



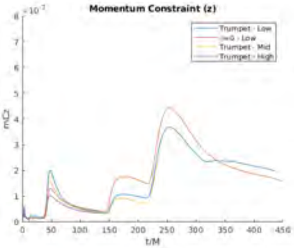


Figure: Deviations from zero of the L2-norm of the y- and z-components of the momentum constraint violation. Dotted lines are simulations using the original $\beta = 0$ initial data, and solid lines use trumpet initial data.

Early $\Psi_4^{2,0}$ and $\Psi_4^{2,2}$ for $q = 1$, $a = 0.8$ Run