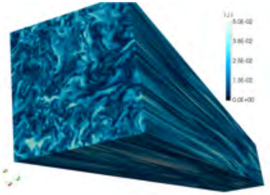

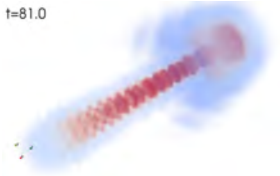

Spectral/Discontinuous Galerkin approach to fully kinetic simulations of plasma turbulence with reduced velocity space

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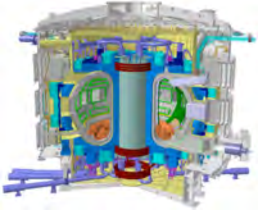
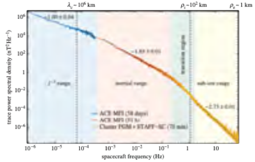
We used Blue Waters to study plasma turbulence

NSF PRAC project #1614664

- Plasma is pervasive in nature and laboratory
- Plasma is often in turbulent state
- Turbulence is hard (a lot of scales)
- Solar wind is the best accessible example of astrophysical plasma turbulence
- The project goal was to study solar wind turbulence numerically in challenging regimes (close to the sun)
- The project is at end
- Next steps — new tools (today's topic)





Kiyani et al., 2015

Vlasov-Maxwell system (VMS)

Microscopic description of collisionless plasmas

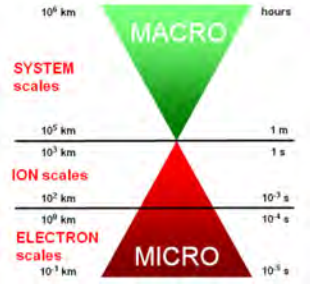
$$\partial_t f_\alpha + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\nabla_{\mathbf{v}} f_\alpha = 0$$

$$\partial_t \mathbf{E} = c\nabla\times\mathbf{B} - 4\pi\mathbf{j}, \qquad \partial_t \mathbf{B} = -c\nabla\times\mathbf{E},$$

$$\nabla\cdot\mathbf{E} = 4\pi\rho, \quad \nabla\cdot\mathbf{B} = 0,$$

$$\rho = \sum_\alpha q_\alpha \int f_\alpha d^3 v, \quad \mathbf{j} = \sum_\alpha \int f_\alpha \mathbf{v} d^3 v,$$

where $f_\alpha = f_\alpha(t, \mathbf{r}, \mathbf{v})$, $\mathbf{E} = \mathbf{E}(t, \mathbf{r})$, $\mathbf{B} = \mathbf{B}(t, \mathbf{r})$.



Parameters of the Earth magnetotail, from *Lapenta, JCP 2012*

VMS is very difficult to solve!

6D + time ⋆ nonlinear ⋆ anisotropic ⋆ multi-scale

Numerical methods for VMS

- **Particle-in-cell (PIC)** — standard method
  - Phase space discretization with macroparticles
  - Simple, robust, statistical noise, low accuracy, mostly explicit
- Eulerian Vlasov solvers
  - Phase space discretization with grid
  - No statistical noise
  - Require a lot of resources: 1000^6 grid points $= 8$ exabyte
- **Transform methods** — focus of this talk
  - Phase space discretization with spectral (moment) expansion
  - Fourier, Hermite basis — *Armstrong et al., 70*
  - Memory requirement/slow convergence might be an issue, but *Schumer & Holloway, 98; Camporeale et al, 06* showed that the Hermite basis can be optimized
  - Major advantage (for AW Hermite and Legendre basis): Naturally bridges between fluid (few number of moments) and kinetic (large number of moments). Optimal way to include microscopic physics in large-scale simulations? (c.f., PIC-MHD coupling)

Spectral plasma solver framework

- Galerkin spectral expansion for velocity space

$$f(t,x,v) = \sum_n C_n(t,x)\Psi_n\left(\frac{v-\alpha}{u}\right),$$

  - Asymmetrically weighted Hermite polynomials
  - Natural fluid(macroscopic)-kinetic(microscopic) coupling
  - Usually small number of DOF is needed
- Discontinuous Galerkin expansion for coordinate space

$$C_n(t,x) = \sum_{l,k} C^l_{k,n}(t)\Phi^l_k(x),$$

  - Very accurate — arbitrary order
  - Shocks and nontrivial geometry
  - Good parallel scaling
- Advance the resulted system with explicit or implicit time integration scheme.
  - Explicit — very fast for some problems
  - Implicit — can skip scales, conserves energy

Spectral plasma solver framework

Other discretizations

- Velocity space

	Fluid coupling	Conservation properties	Stability
AW Hermite	☺	☺	☹
SW Hermite	☹	☹	☺
Legendre	☺	☺/ ☹	☺/ ☹

- Coordinate space
  - Pseudo spectral method based on Fourier modes
    - ⋆ More than perfect when you fit into one node
  - **Discontineous Galerkin**
    - ⋆ Great so far — perfect scalability, mass/energy conservation, arbitrary order, somewhat heavy with small CFL
  - Finite volume
    - ⋆ Similar to DG, but with different compromises

Spectral plasma solver framework

Note on implementation

DG-Hermite decomposition of full Vlasov-Maxwell system:

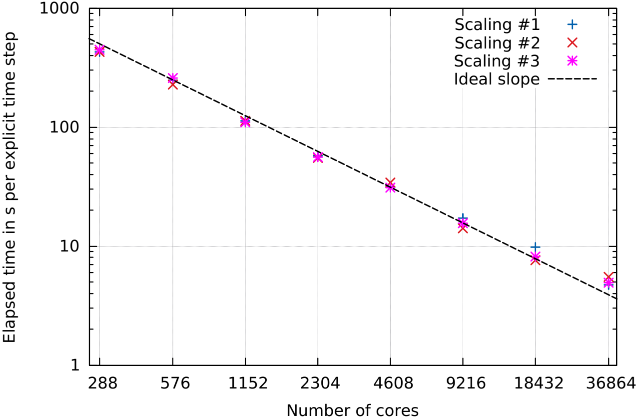
- Distributed memory based — 3D3V SPS-MPI via PETSc
- Explicit time discretization (Family of different Runge-Kutta's)
- Implicit time discretization (conserves energy — implicit mid point)
  - Non-linear solver: JFNK + GMRES/BCGS
  - Preconditioning (PILU from Hyper, Block Jacobi) — memory optimization is in progress

Other approaches to space discretization

- Fourier + Hermite (efficient openMP based 2D3V and MPI based 3D3V)
- Finite volume/difference + Hermite
- Legendre: Vlasov Poisson system (proof of concepts)
- SW Hermite: Vlasov Poisson system (proof of concepts)

Parallel efficiency

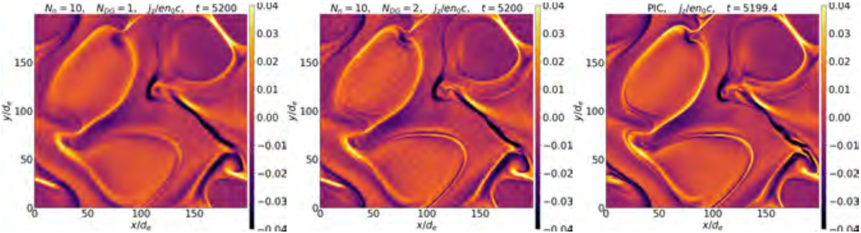
- 2D3V Orszag-Tang vortex test with explicit time integrator — scales very well up to 50000 DOF per core



Example

2D Plasma turbulence: Orszag-Tang vortex

- Excite two large scale flow vortices and let them evolve to form small scale structures

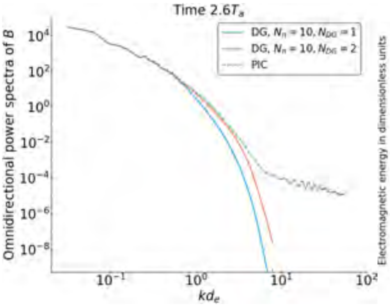


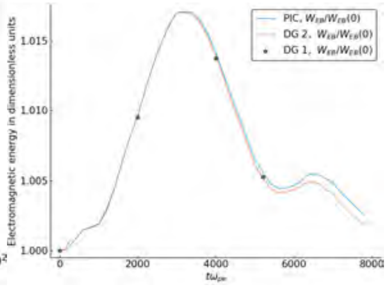
- SPS resolution: $N_x N_y = 512^2$, $N_{vx} N_{vy} N_{vz} = 10^3$
- PIC resolution: $N_x N_y = 3520^2$, $N_p = 4000$

Orszag-Tang vortex test

Spectrum and energy

- Omnidirectional spectrum of magnetic field fluctuations
  - SPS is noiseless, but has numerical diffusion when spatial resolution is not sufficient
- Electromagnetic energy is consistent even with reduced velocity space





Conclusion

- Spectral Plasma Solver (SPS) is a unique framework to study kinetic multi-scale plasma physics problems
  - Built-in fluid/kinetic coupling is efficient way to incorporate microscopic physics
  - Reduced velocity space is able to reproduce important microscopic physics
  - Mass, and energy conserving — accurate long time integration
  - Flexible time discretization — implicit or explicit as needed
  - Flexible spatial discretization (nontrivial boundary conditions)
  - Great parallel scalability (c.f. pure spectral methods)