THE WELLSPRING OF ALL PHASES ON THE KAGOME LATTICE

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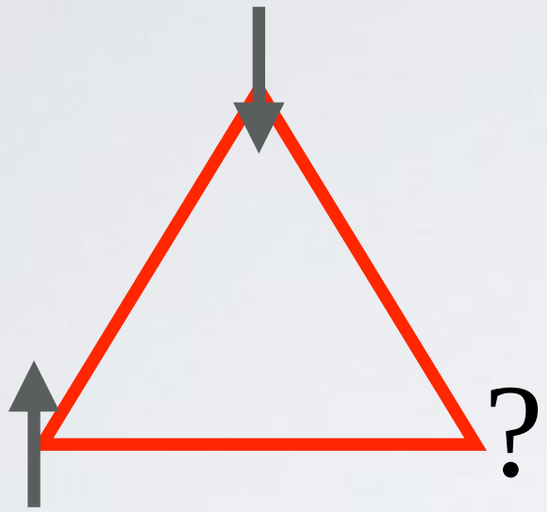
with Hitesh Changlani, Dmitrii Kochkov, Krishna Kumar, Eduardo Fradkin

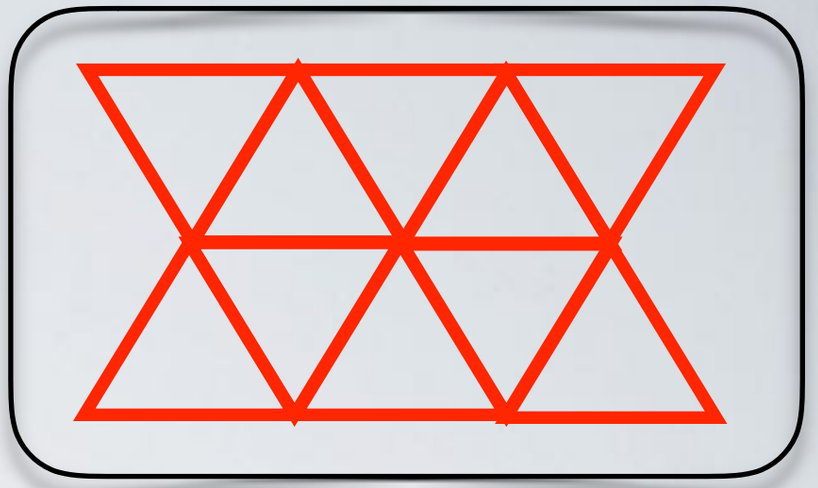

Blue Waters Symposium Talk

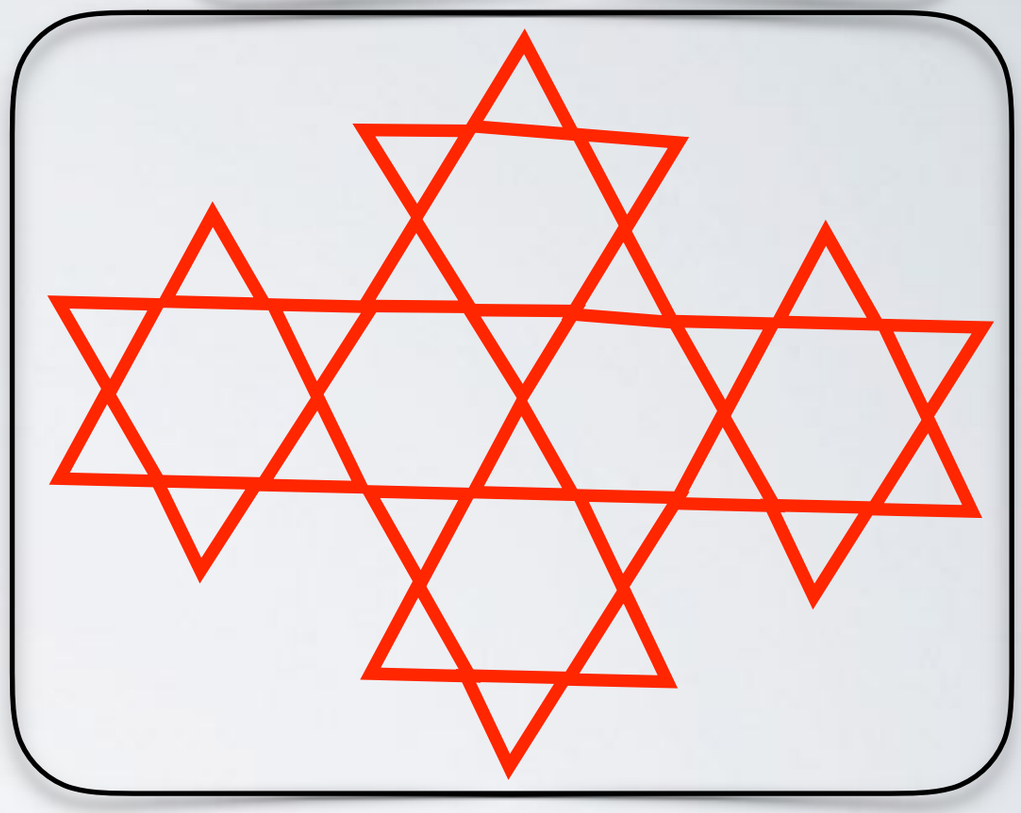
Frustrated Quantum Magnets.....

- Insulator - Electrons don't move
- Interaction between electron spins
  - spins want to anti-align



- Frustration from triangles





Quantum Materials:

Herbertsmithite

Kapellasite

Vesigniette

Volborthite

Quantum Mechanics is hard

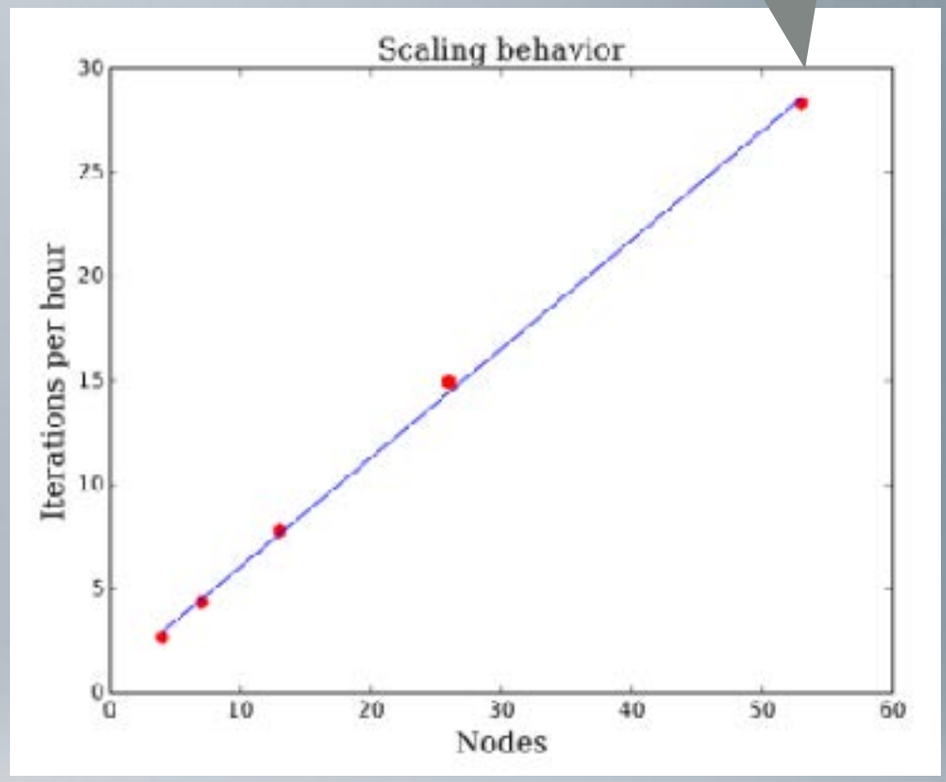
Quantum Mechanics is computationally hard

There is an exponential scaling

~50 years of numerics in quantum mechanics

Algorithm: Find lowest eigenvector of $2^n \times 2^n$ matrix

Even at this scaling, if we wanted to do 50 spins, would need one million nodes for a day.



Today:

State of the art: 36 spins (many calculations)
~8 hours on Blue Waters

50 spins (1 calculation)
(Lauchli on German supercomputers)

The quantum mechanical problem hits a wall even with Blue Waters scale.

Approximate algorithms or better exponential scaling a route forward.

Blue Waters still critical. This is an exploratory and iterative science. The story I'm going to tell involved calculations we didn't know we were going to be doing at the beginning of the project. Thousands of iterative simulations needed.

What we found?

Hamiltoniain (i.e. matrix) with exponentially many ground states.

Why it's interesting?

Each ground state represents a phase of matter (liquid, solid, gas, anti-ferromagnet, etc)

This means that there is a special Hamiltonian where all phases meet.
This means that it sources all interesting phases on a class of materials.

Including a particularly interesting (and useful for quantum computing) phase: a spin-liquid.

In strongly correlated systems like frustrated magnets...
there are a menagerie of competing phases.

Z2 (or Dirac) Spin Liquid

Chiral Spin Liquid

q=0 magnetic order

$\sqrt{3} \times \sqrt{3}$ order

Ferromagnetism

...

Heisenberg (White/Huse)

2/3 Plateau (this work)
1/3 Plateau (Donna Sheng)
Chiral Term (Bela Bauer, Andreas Ludwig)
J1, J2, J3 (Donna Sheng)

Is this a cosmic coincident or is their a deep reason behind this?

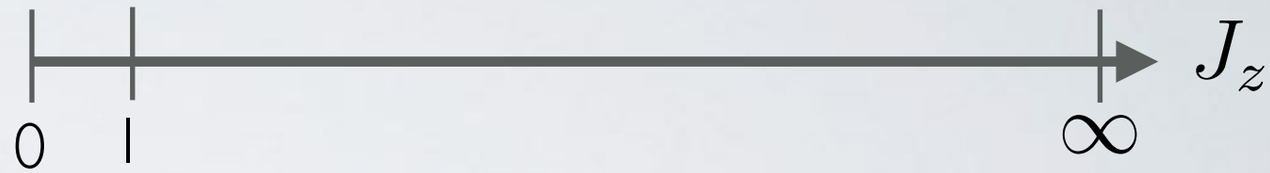
Aside: Spin Liquids are really interesting.

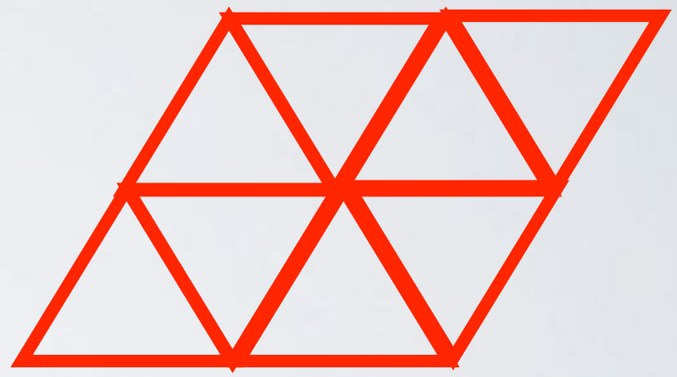
Phil Anderson suggested that the n.n. Heisenberg model on the triangular lattice wasn't a neel state (frustration!)

RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

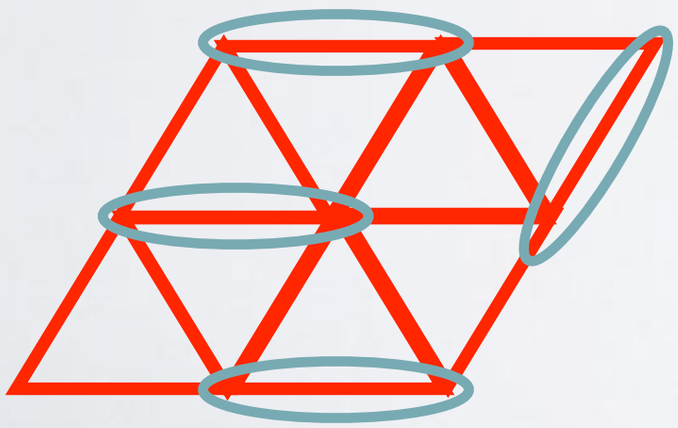
(Received December 5, 1972; Invited**)





instead, he suggested it was a RVB state.
(today we would call such a thing a spin-liquid).

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



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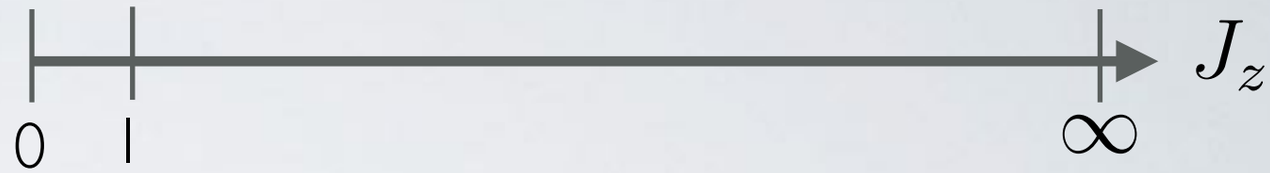

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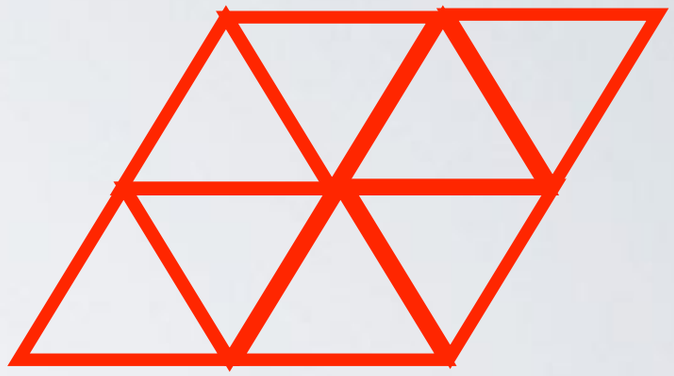
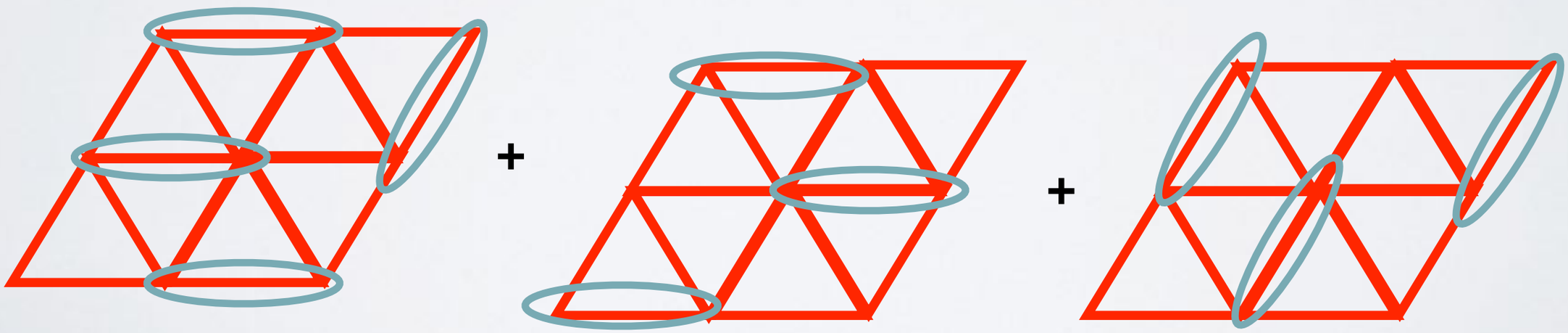


$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



Like Benzene

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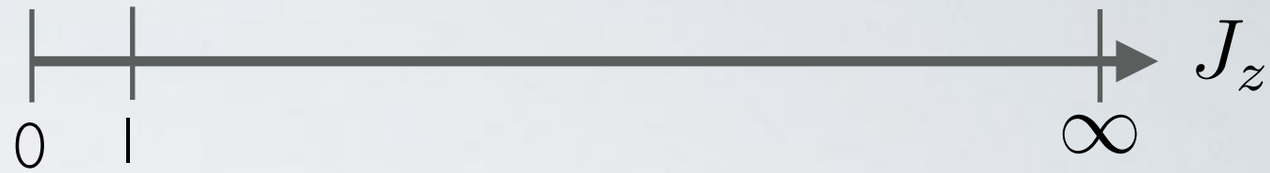

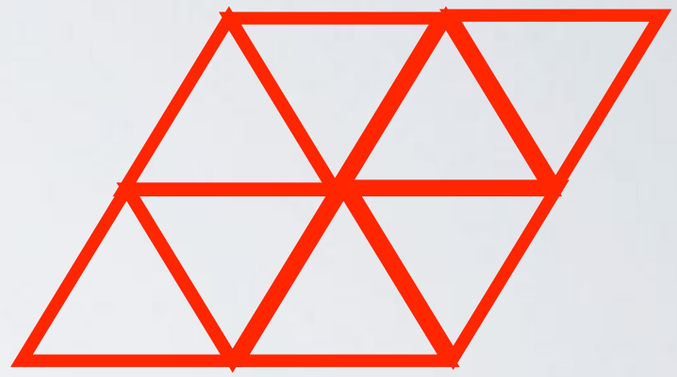

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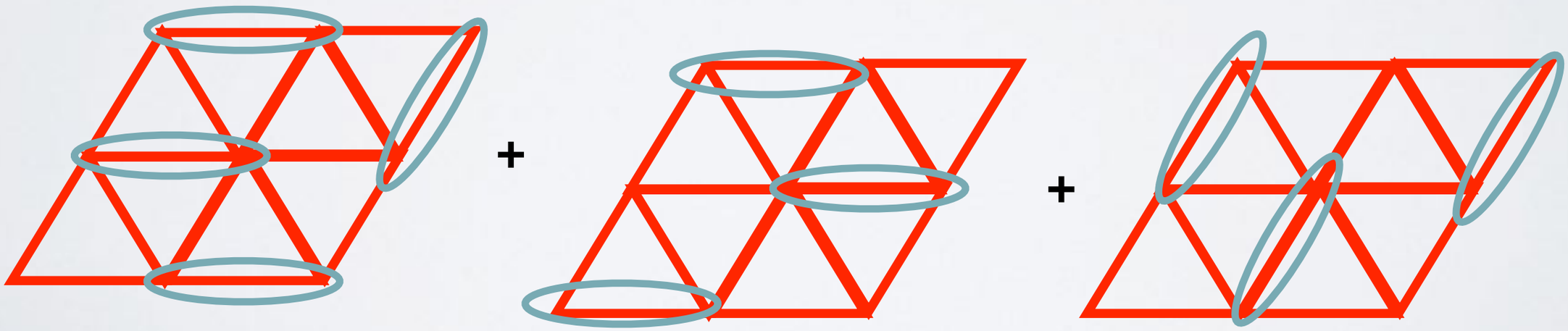
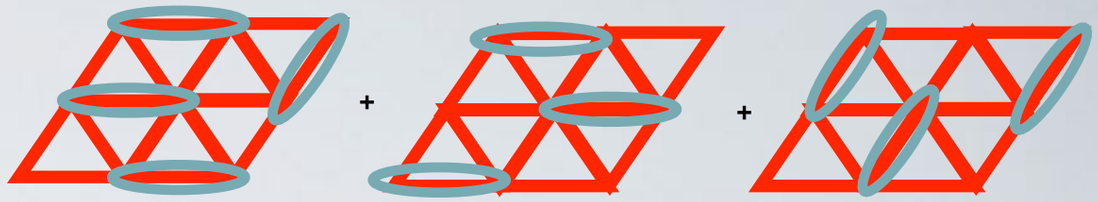


Not actually a spin liquid - years of numerics (1970-1990)

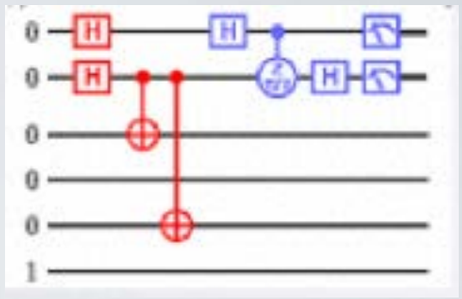
Like Benzene

Aside: Spin Liquids are really interesting.



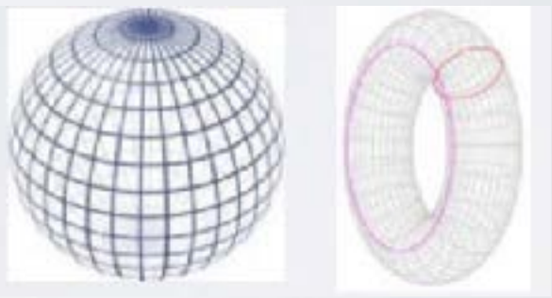
Beyond the Landau theory of phases - no broken symmetries!

Long Range Entanglement - Can't be produced from a product state via a short quantum circuit



Fractionalized Excitations - Electron breaks into multiple emergent pieces.

Topological Degeneracy - Manifold dependent geometry

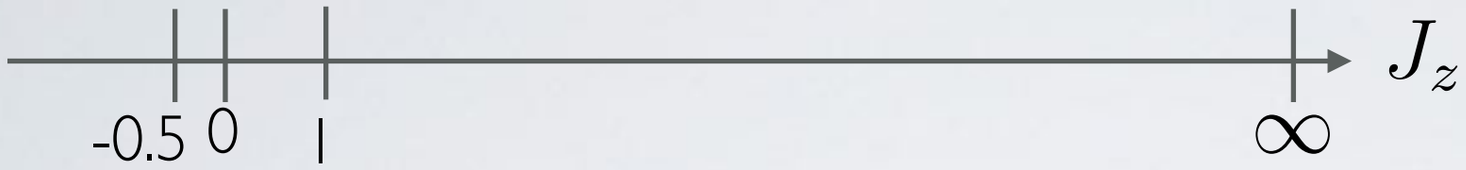


The search for spin liquids is truly a hunt. We haven't had any good story for what sort of lattices should support spin liquids.

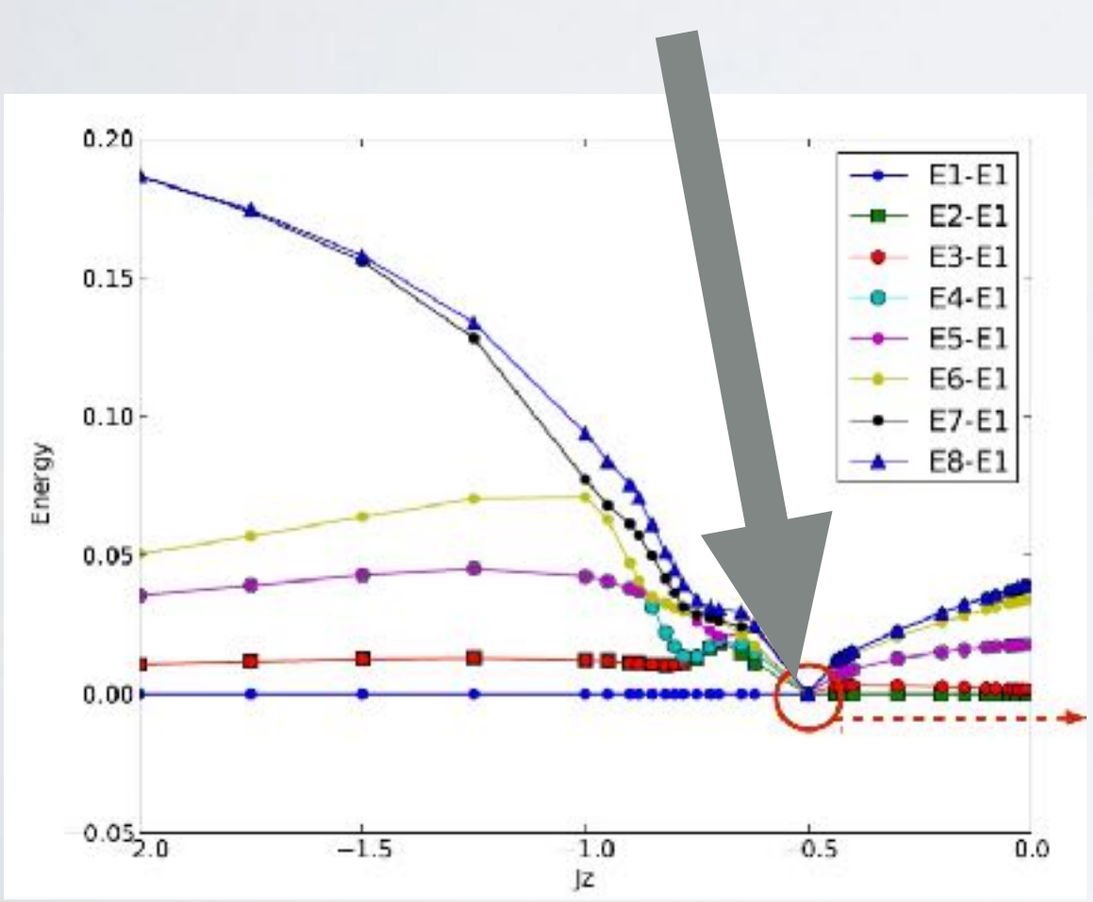
The hunt for **spin liquids** is one of the forefront areas of condensed matter research!
Would be useful for storing quantum information and topological quantum computing!

An interesting discovery.... (amazing it hasn't been known for 30 years)

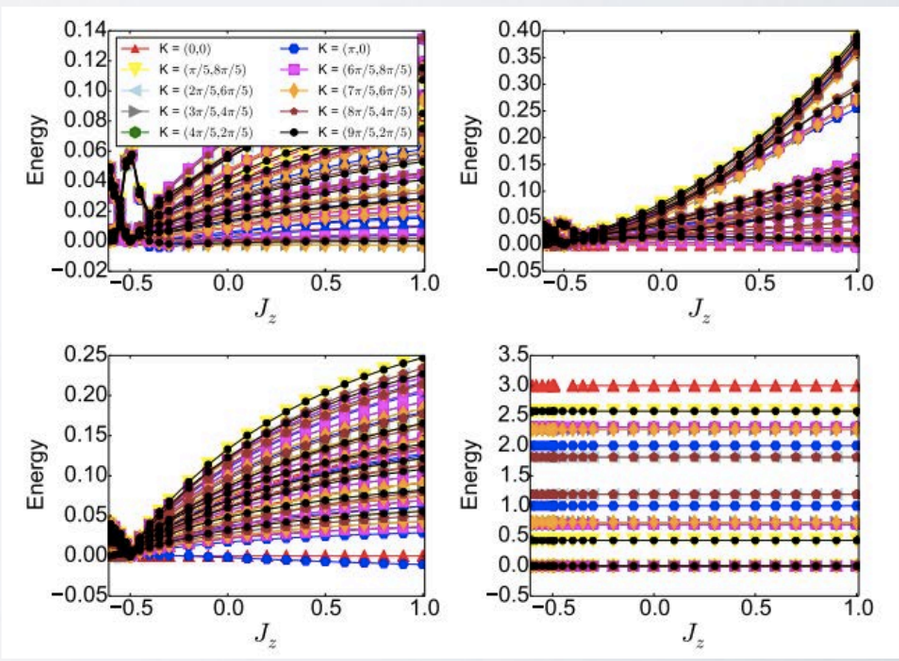
$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$



On the kagome:
massive exact degeneracy in the XXZ model!
exactly -J/4

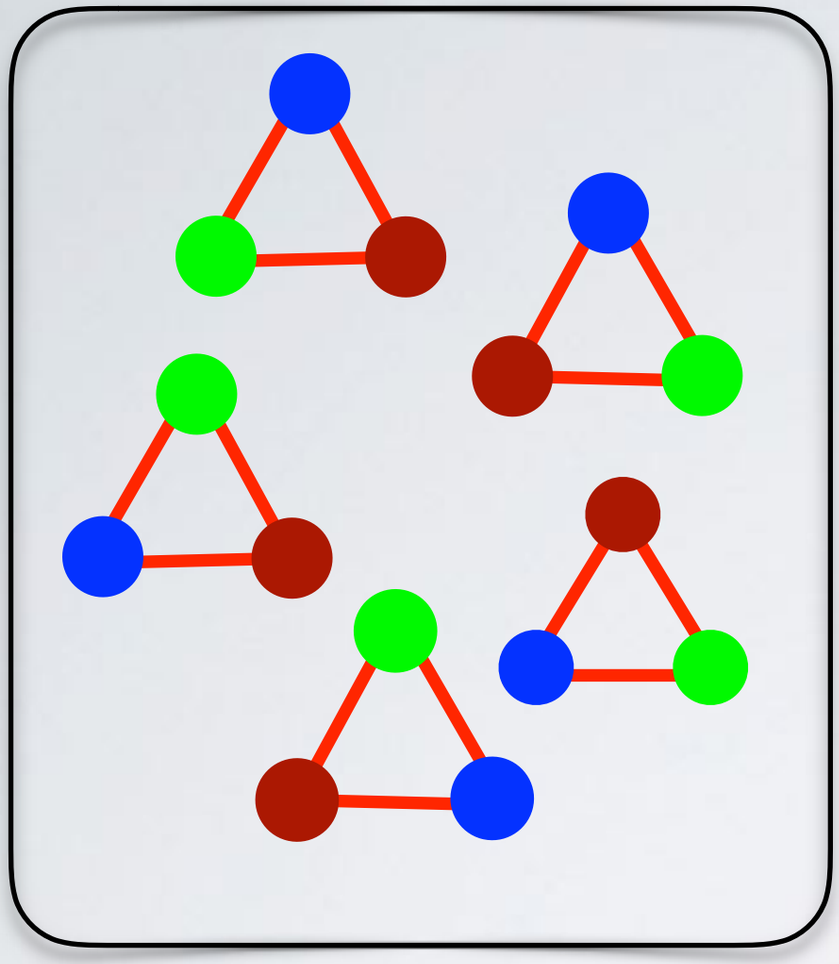


Finding this was only possible because we could afford to explore many points on Blue Waters.



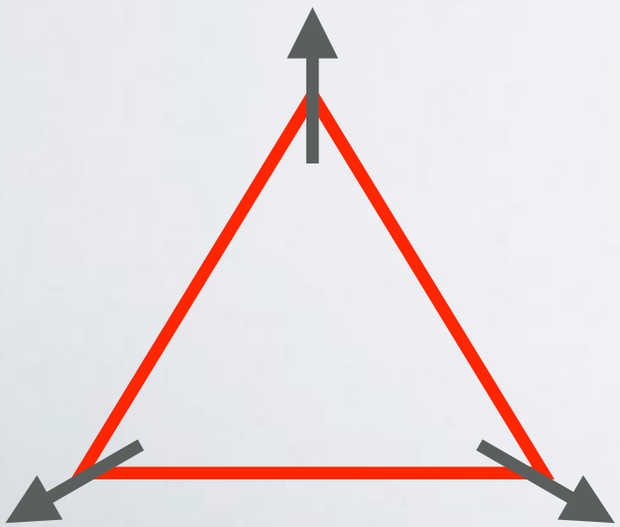
What's going on? Who ordered this?

Who ordered that?

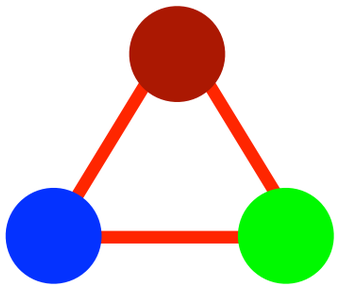


$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$

These are all the ground states of a single triangle.



Define
3 “colors”



$$|a\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

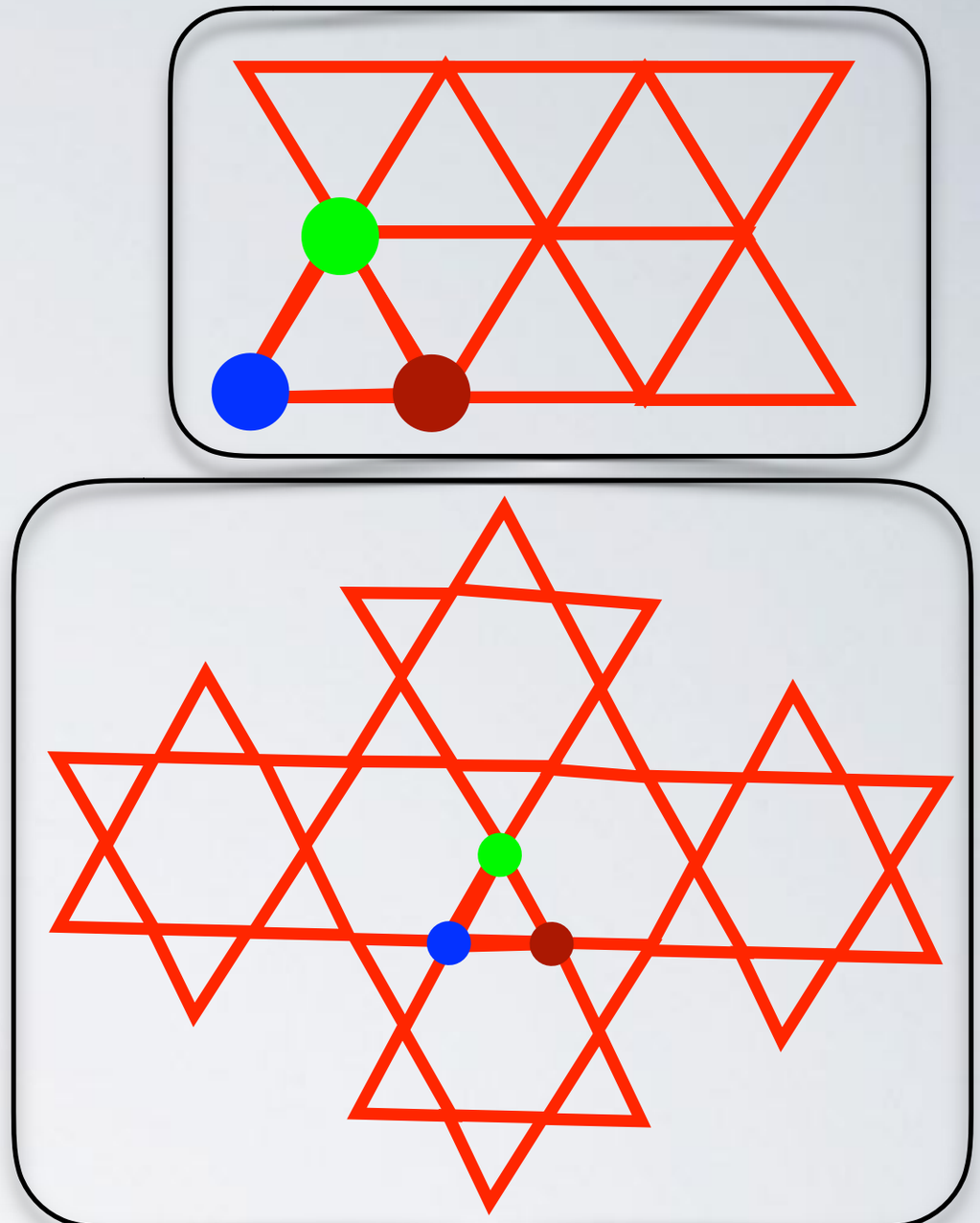
$$|b\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega|\downarrow\rangle\right)$$

$$|c\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega^2|\downarrow\rangle\right)$$

What about many triangles?

$$H = \sum_{\Delta} H_{XXZ0}$$

Paste together ground states over individual triangles



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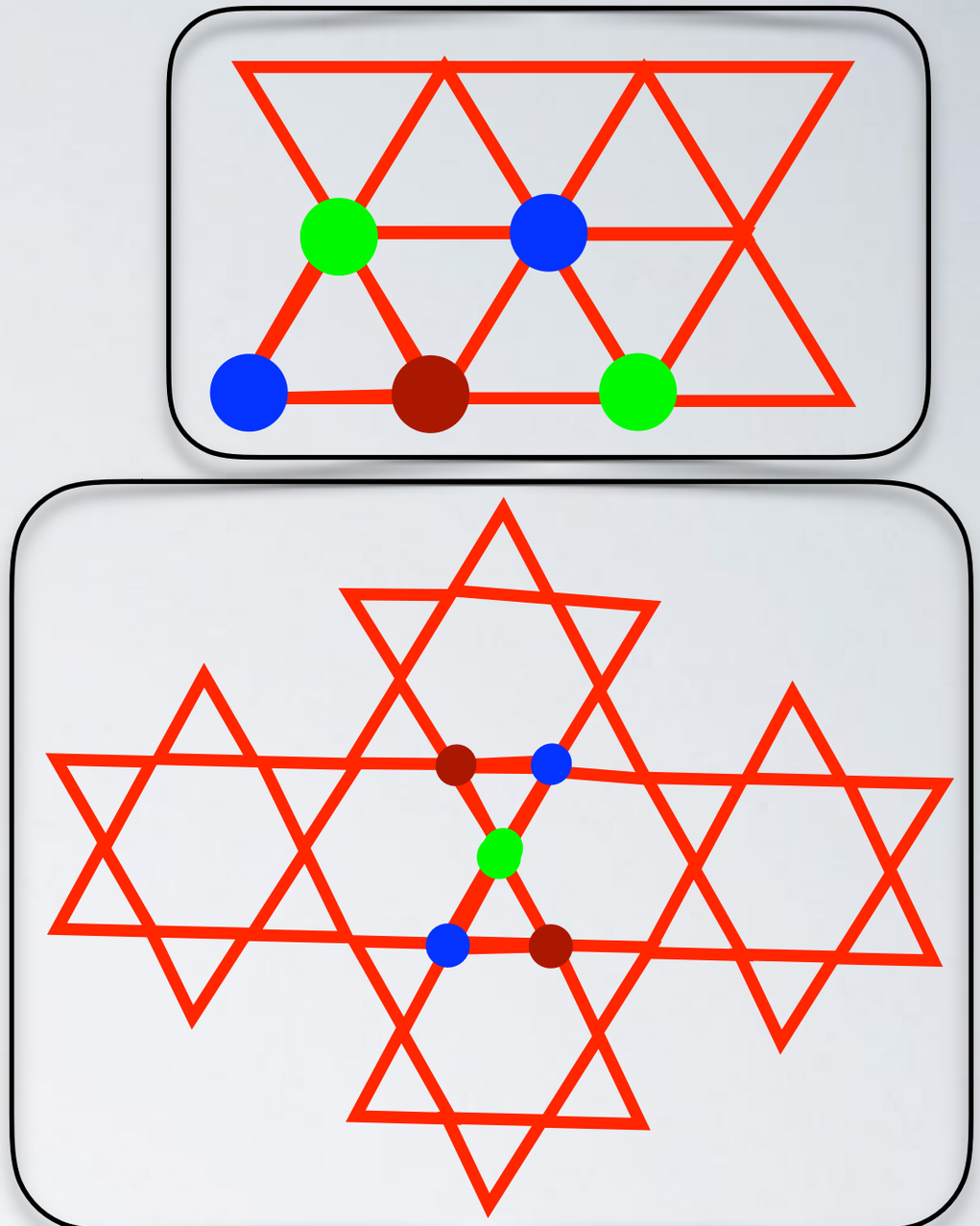

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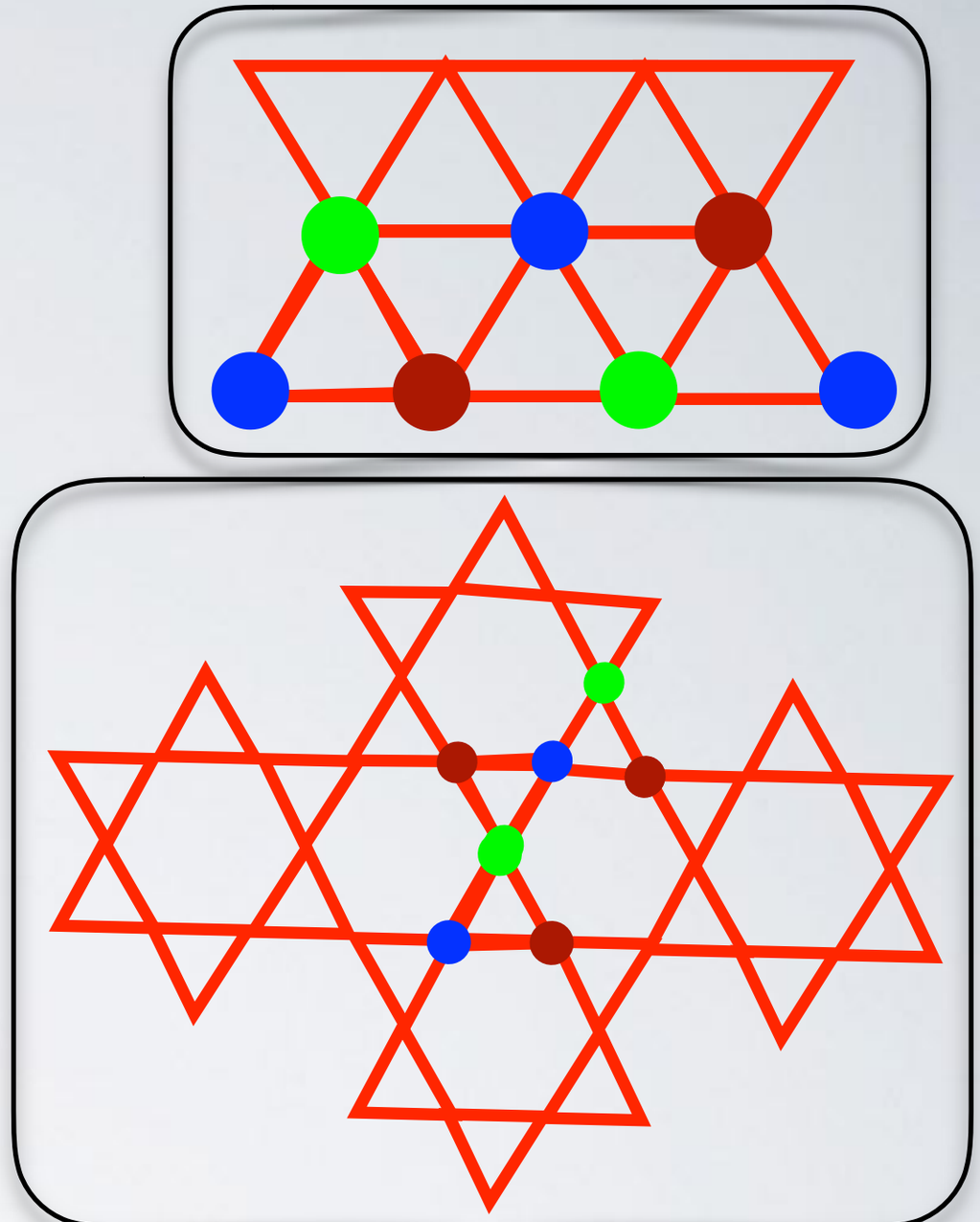
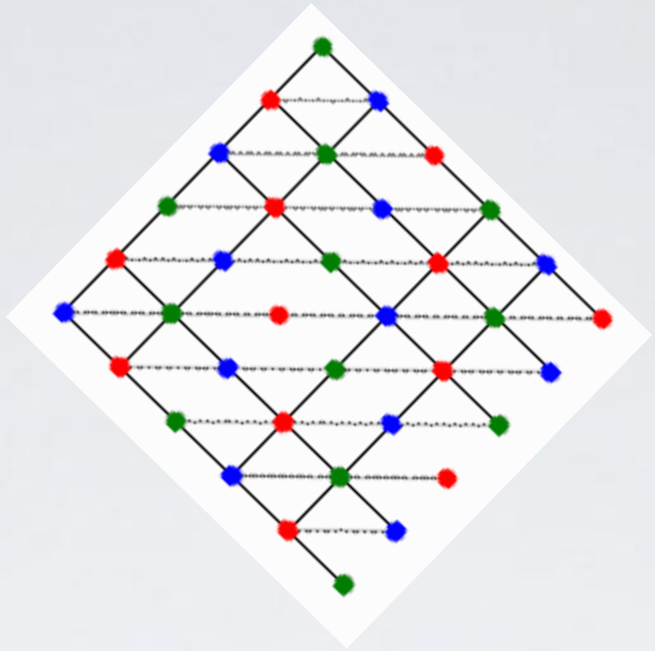
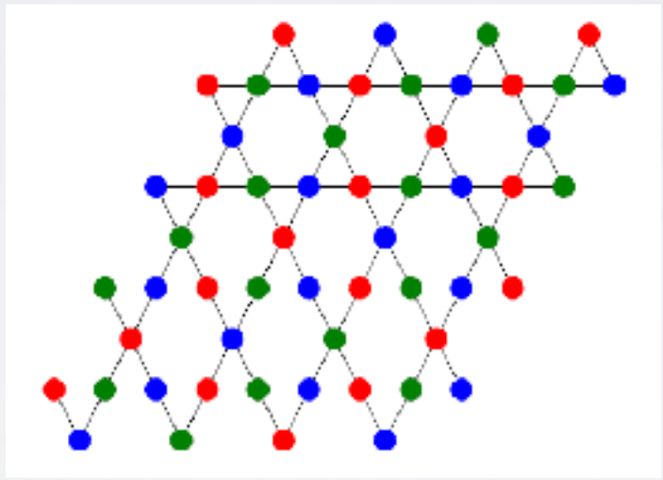


How many colorings?

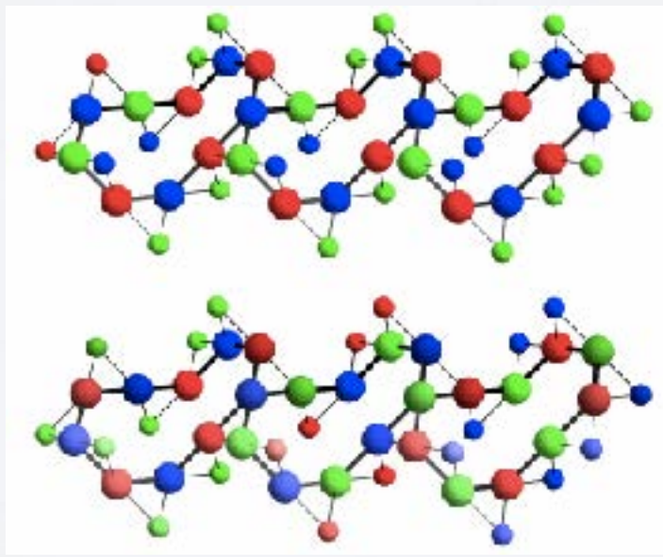
One coloring:



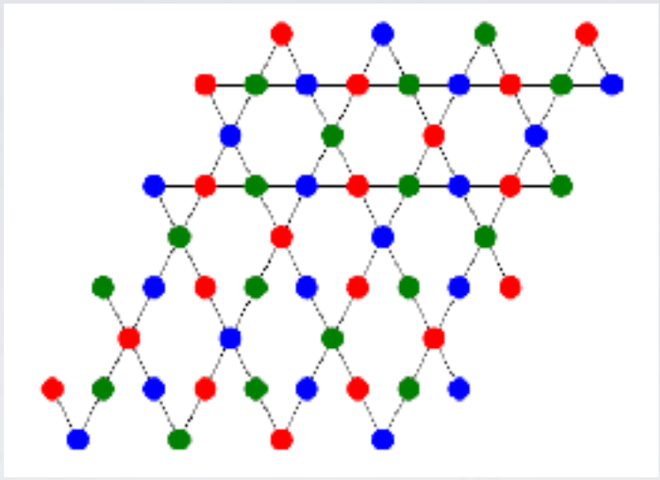
Many colorings:

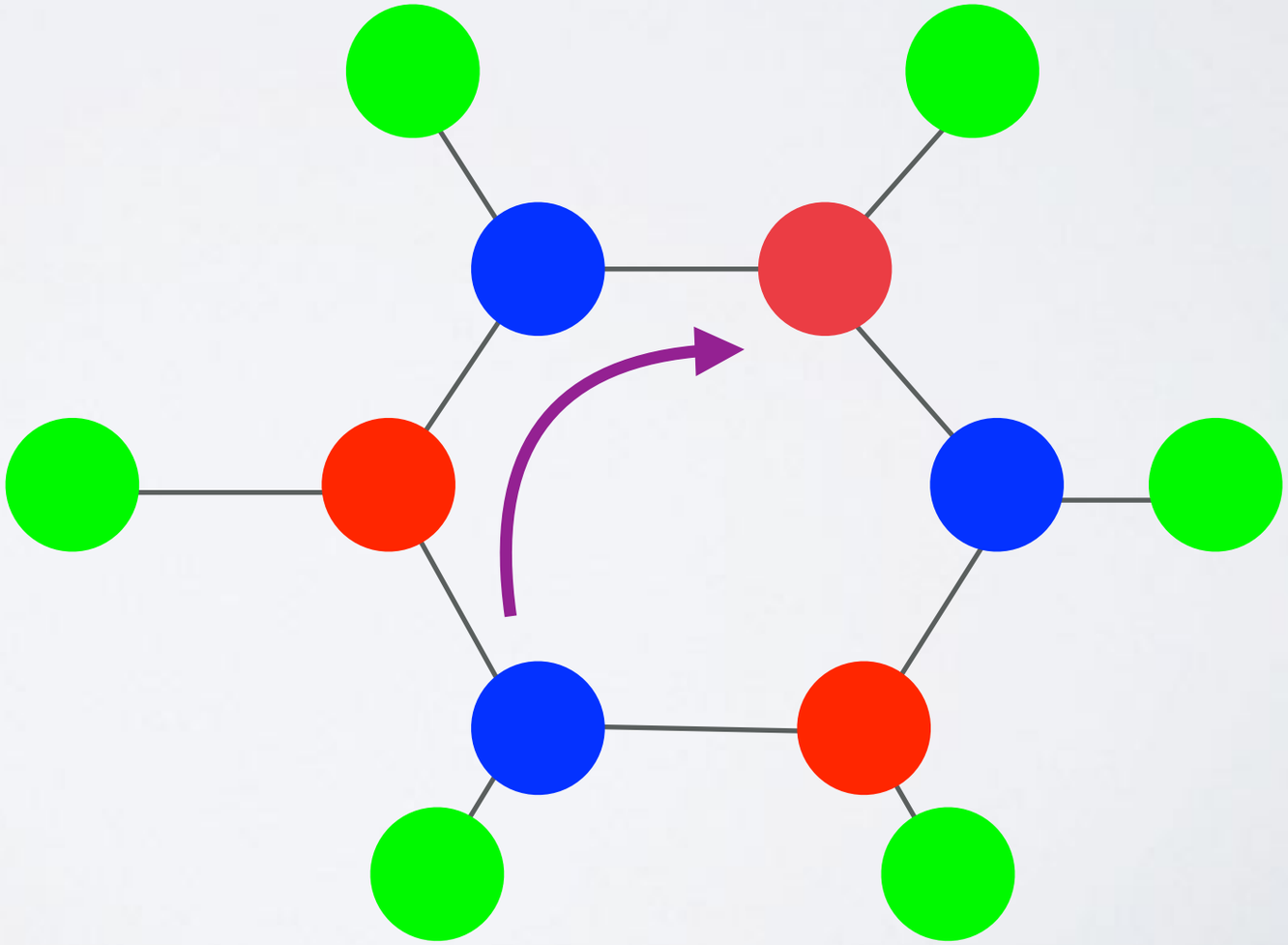


Many colorings:



Many colorings:





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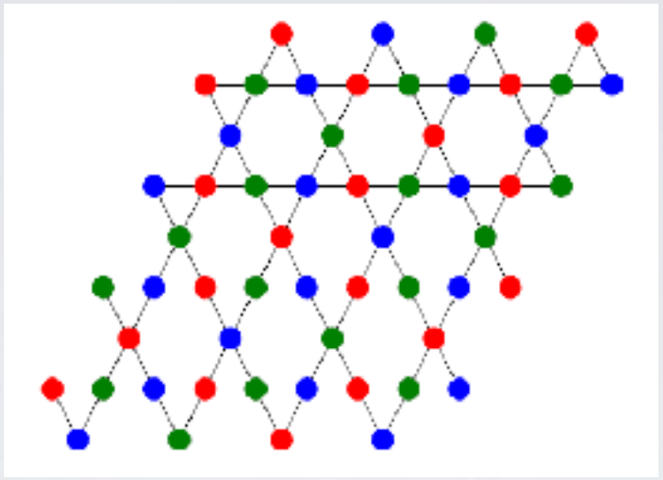
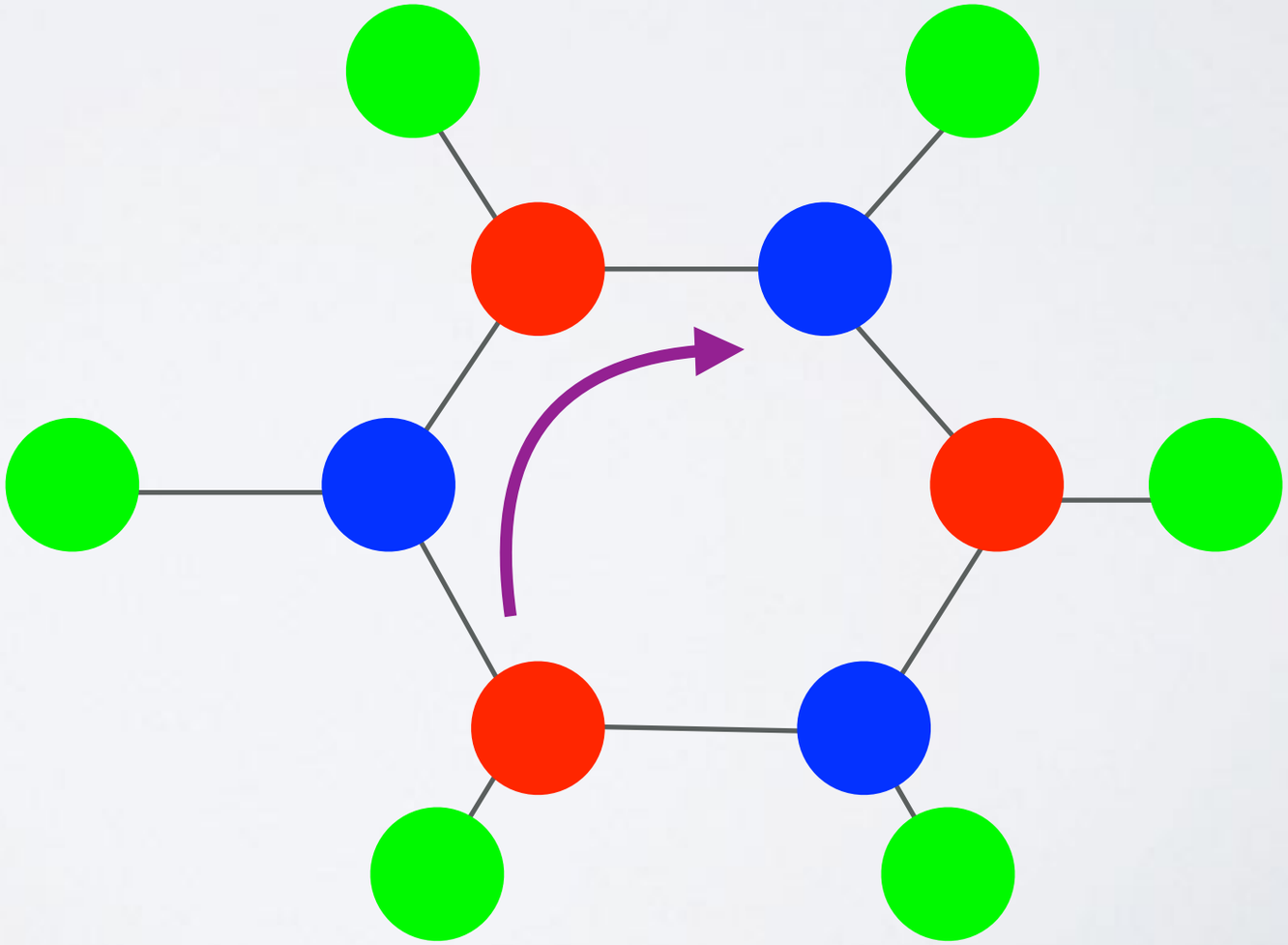
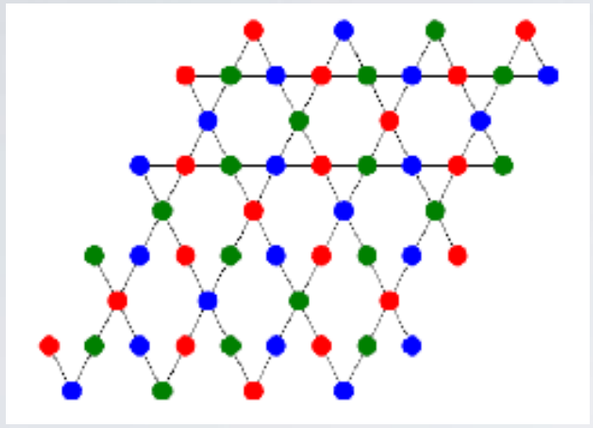




Can we use this to understand the kagome lattice?

Many colorings:



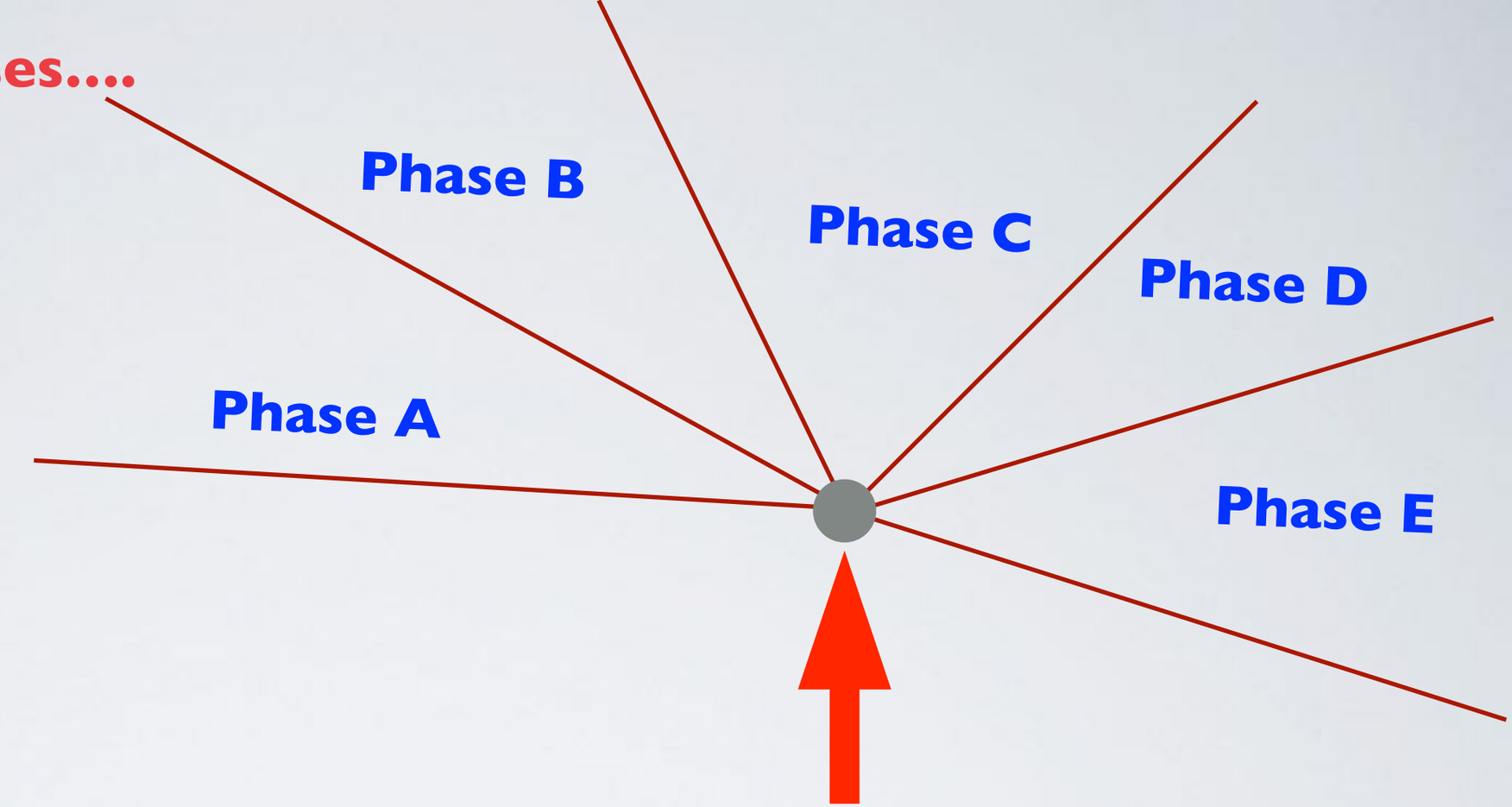
An exponential number of colorings!

1.208^N (from Baxter)

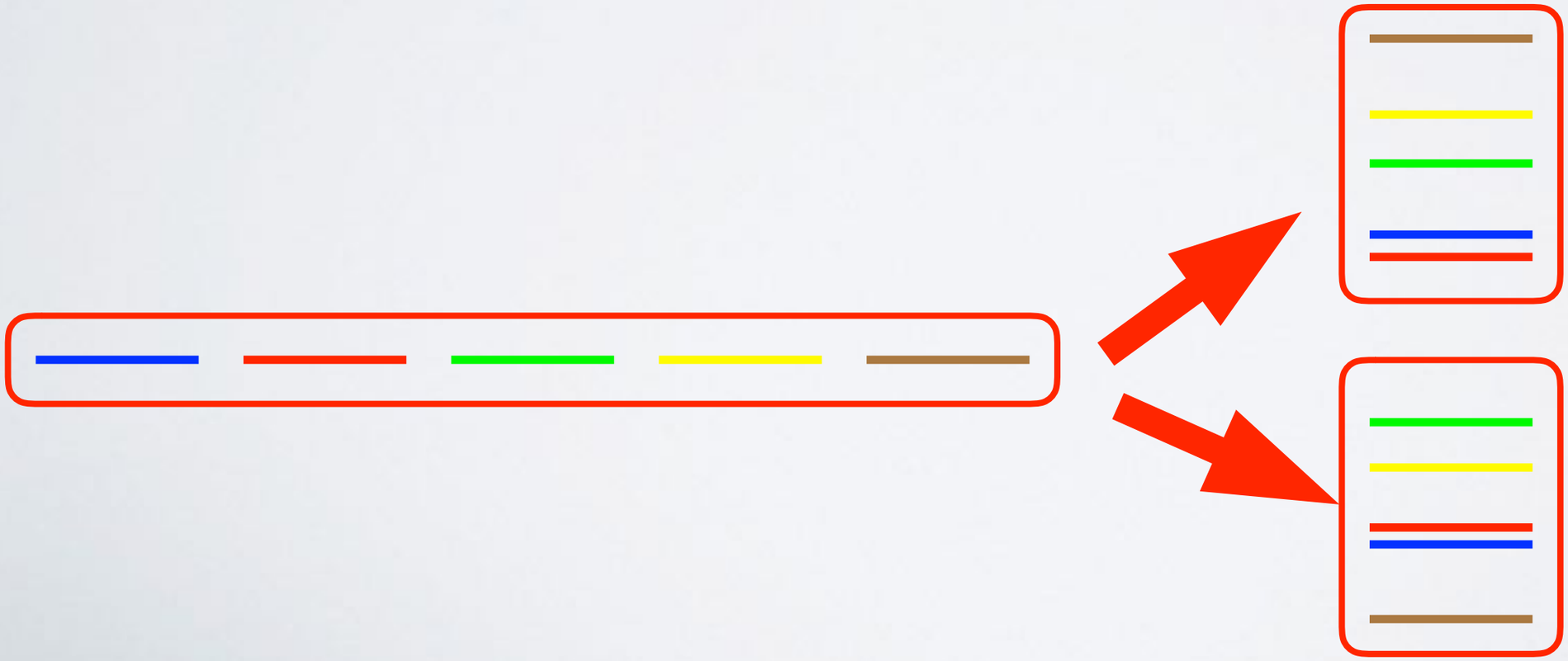
But much fewer then Ising configurations….

Lattice	Ising configs	Colorings
2x2x3	924	8
3x2x3	48620	16
4x2x3	2.7 million	32

Connect to known phases....

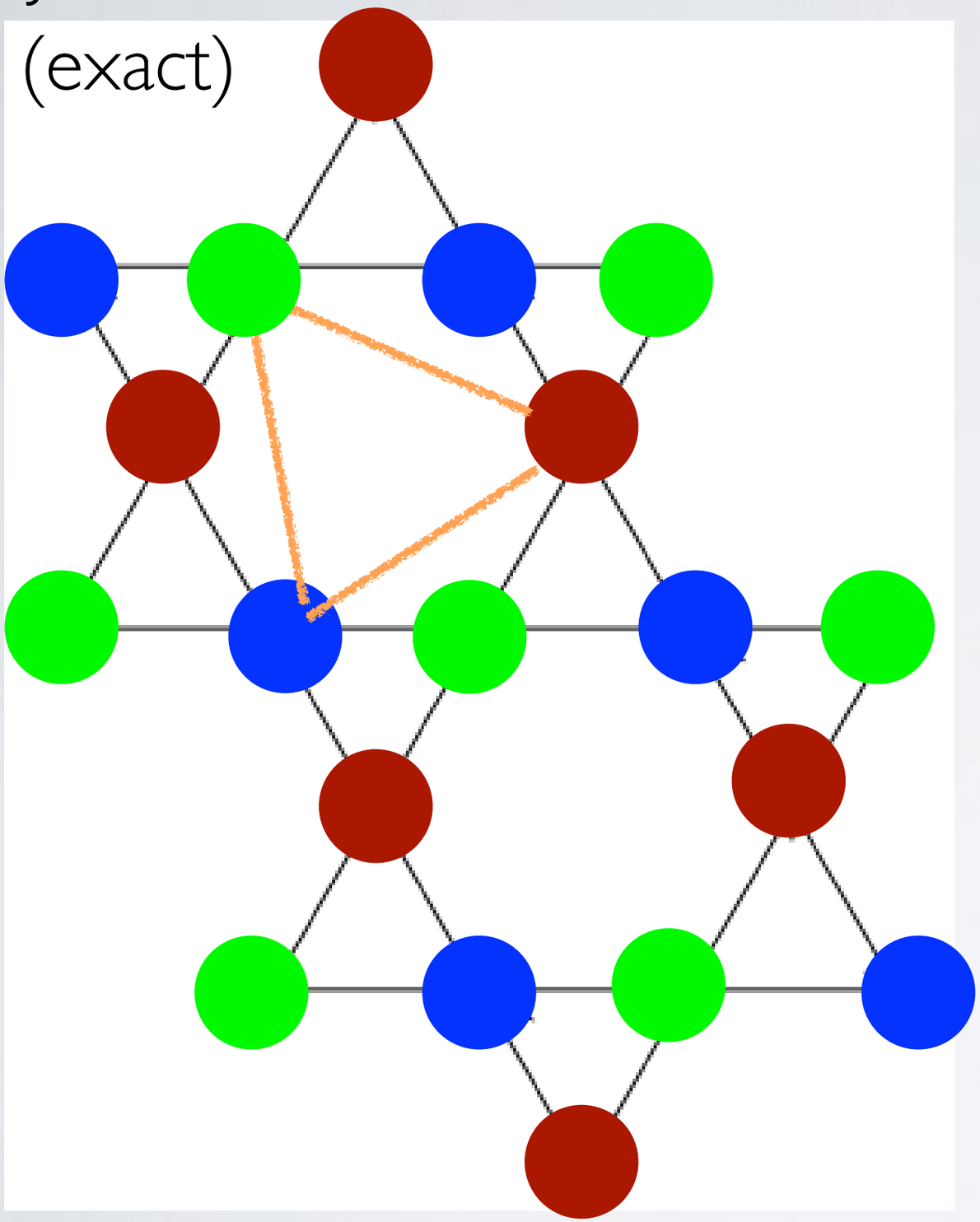


The mother of all phases?



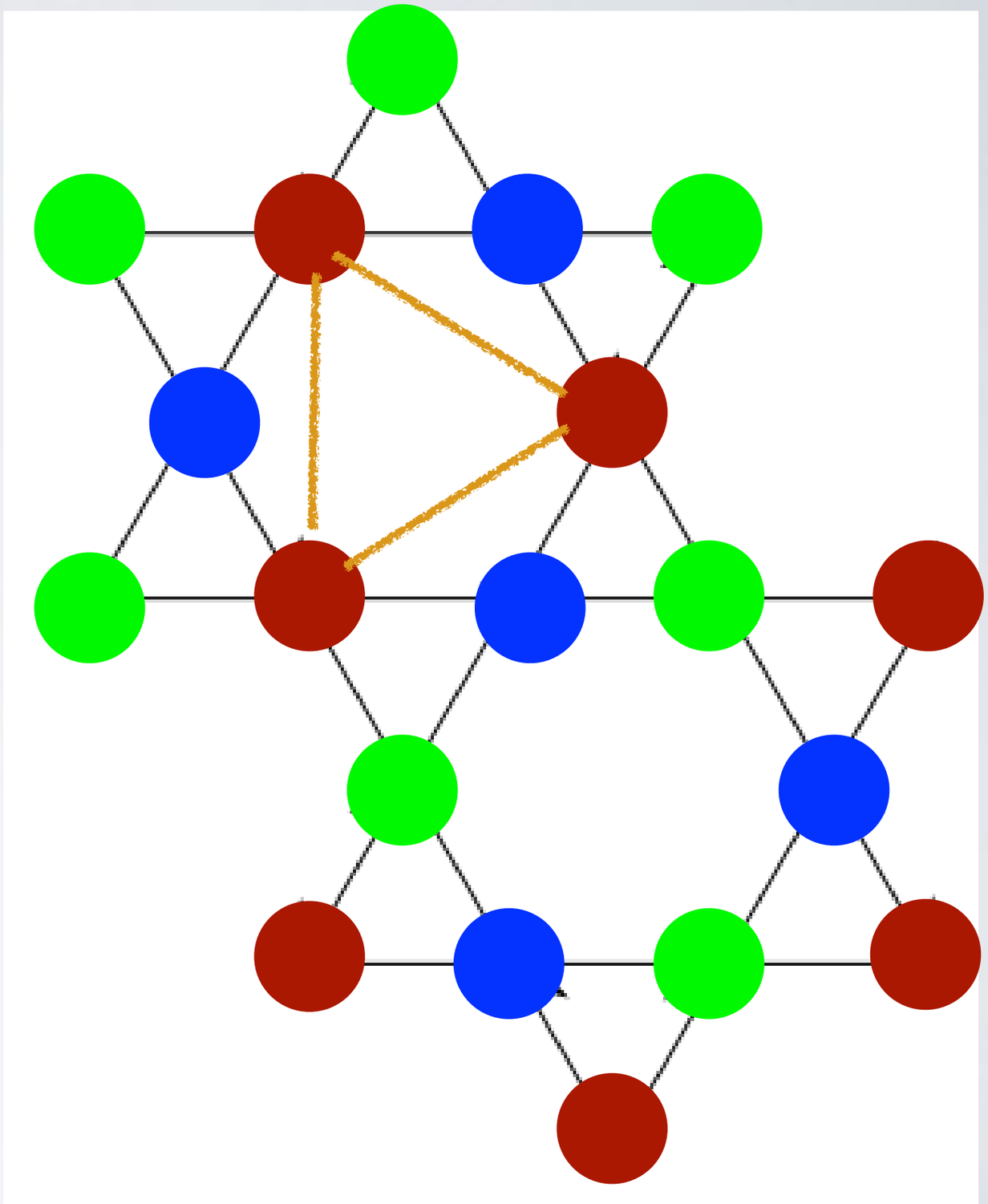
J2>0

(exact)

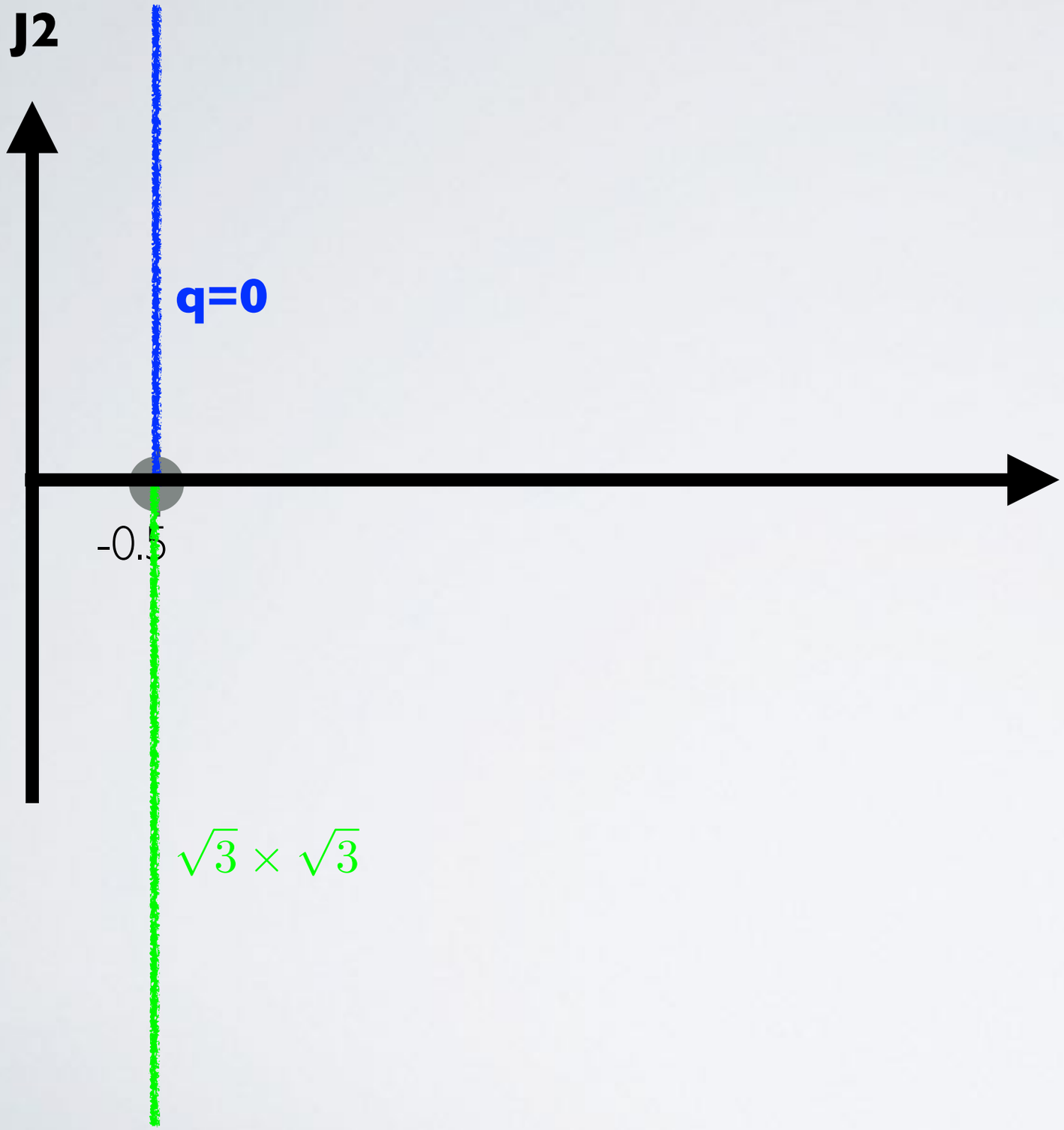


q=0

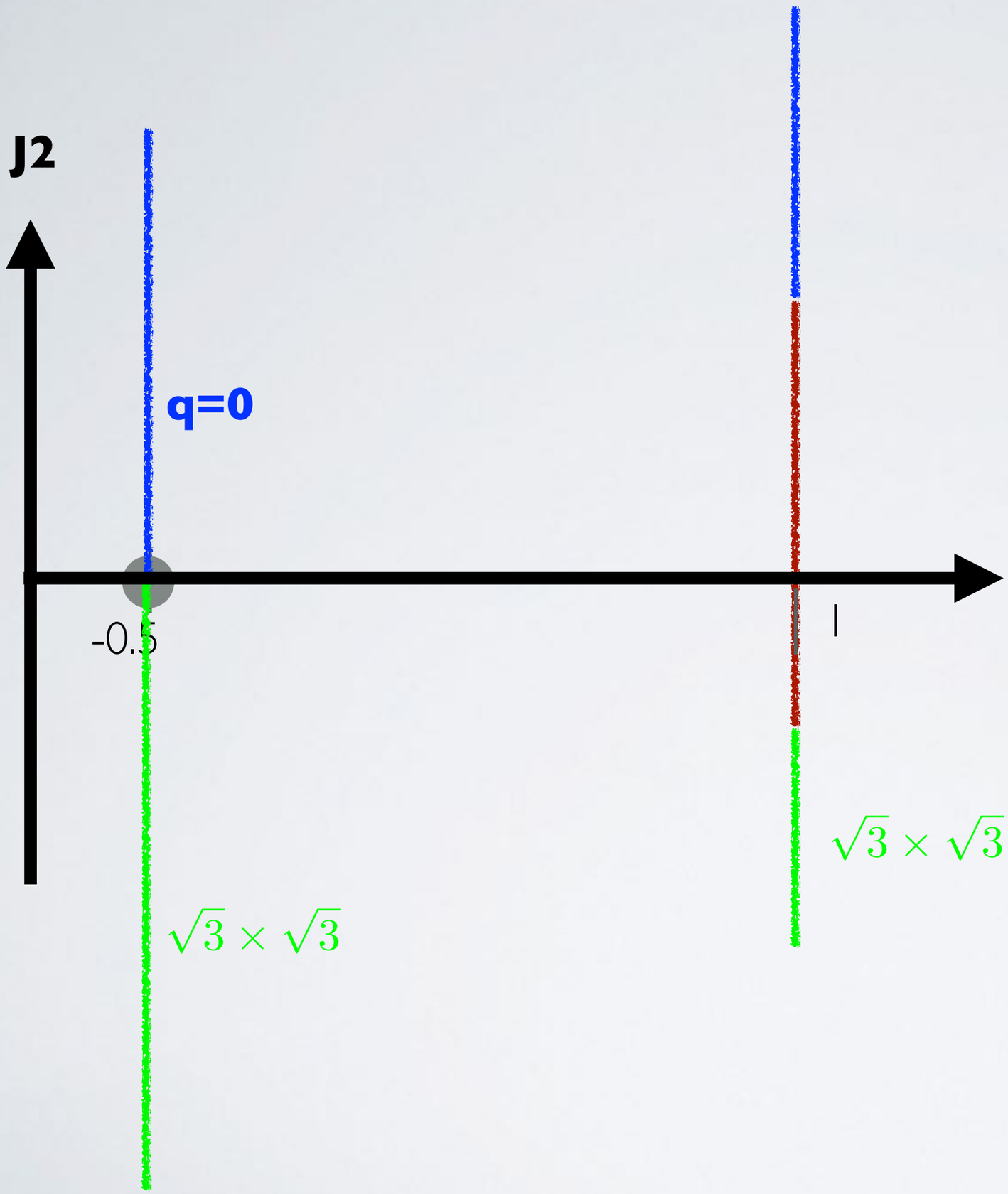
J2<0



$\sqrt{3} \times \sqrt{3}$


J2
q=0
-0.5
$\sqrt{3} \times \sqrt{3}$

$S_z = 0$



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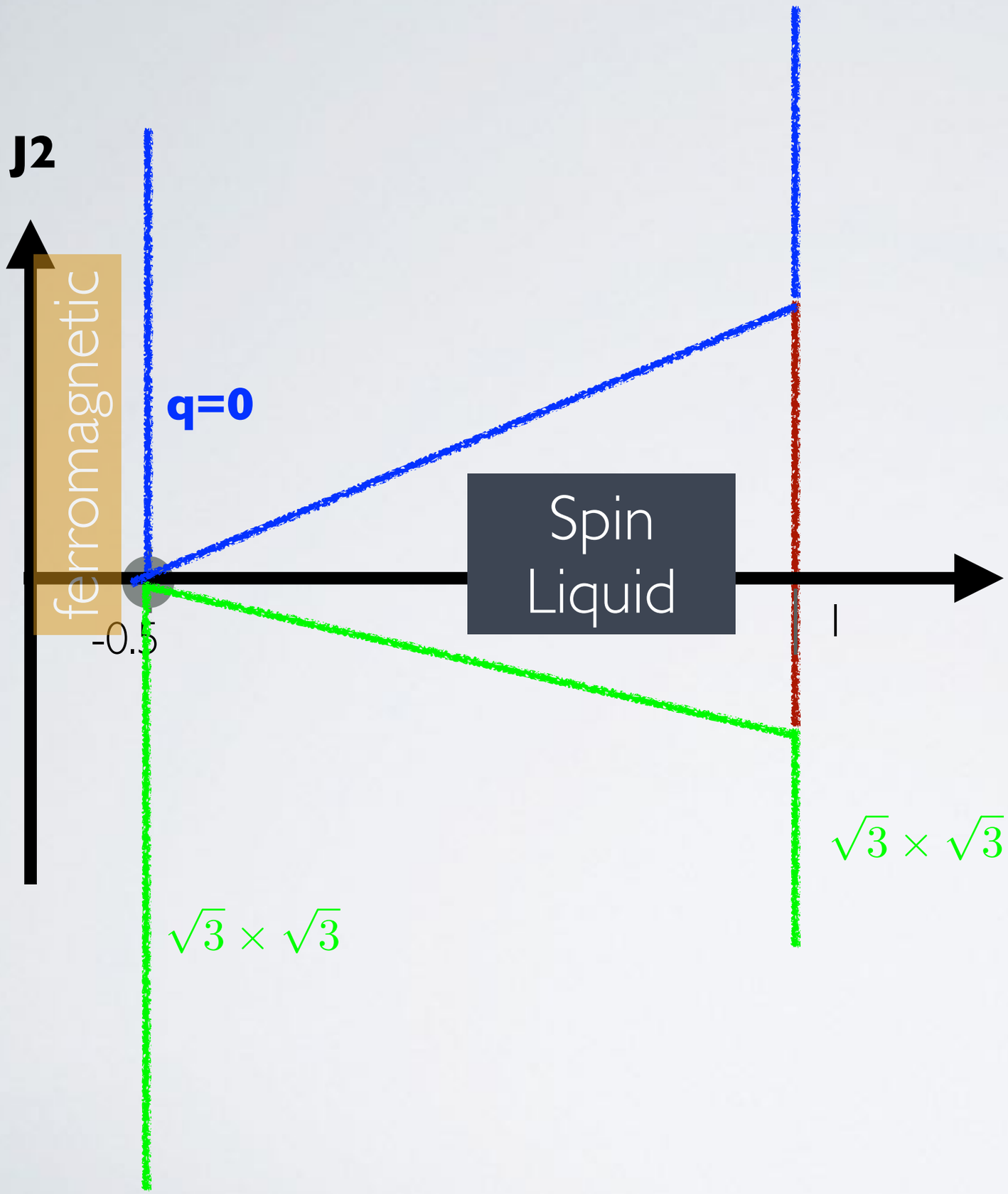
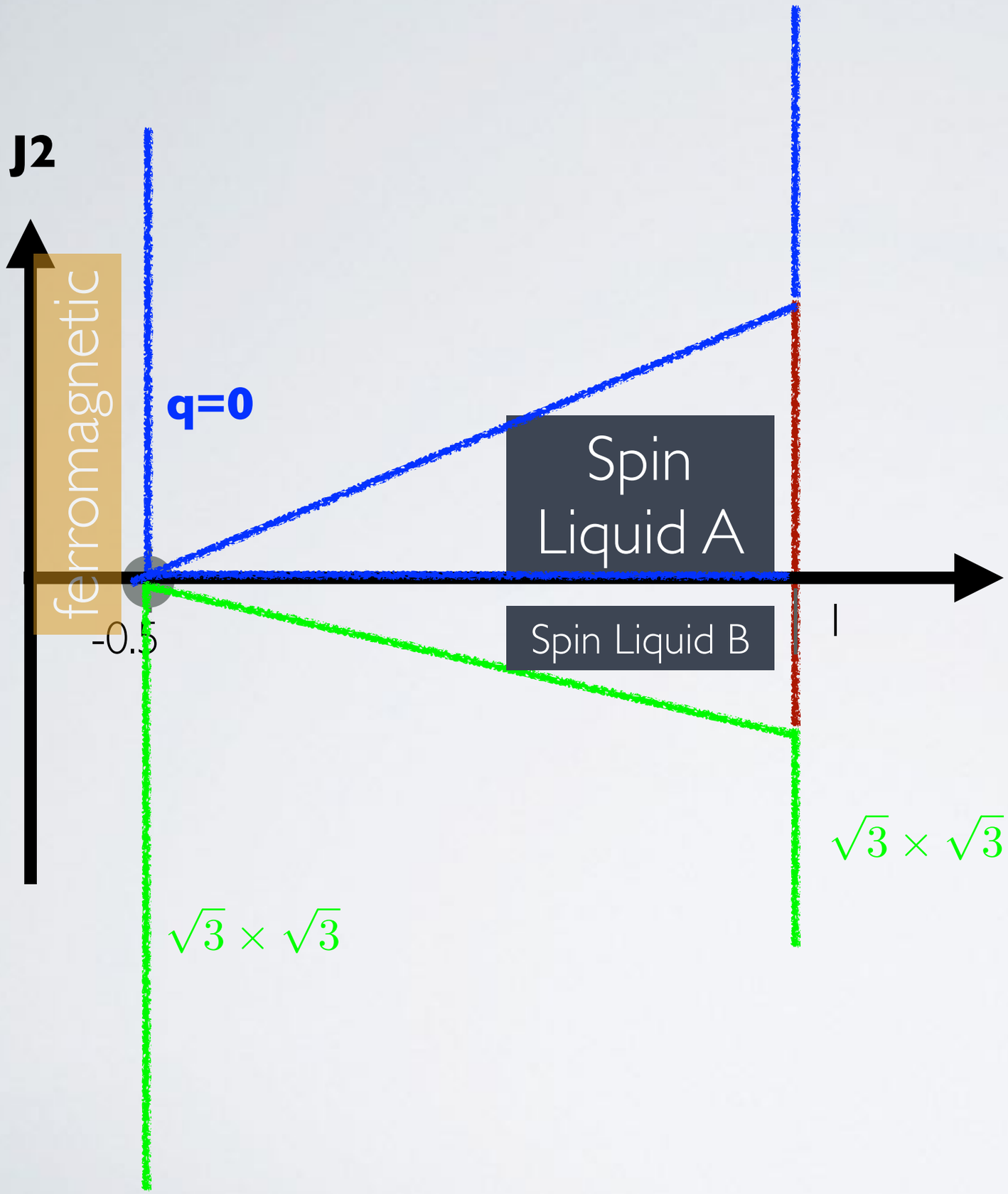
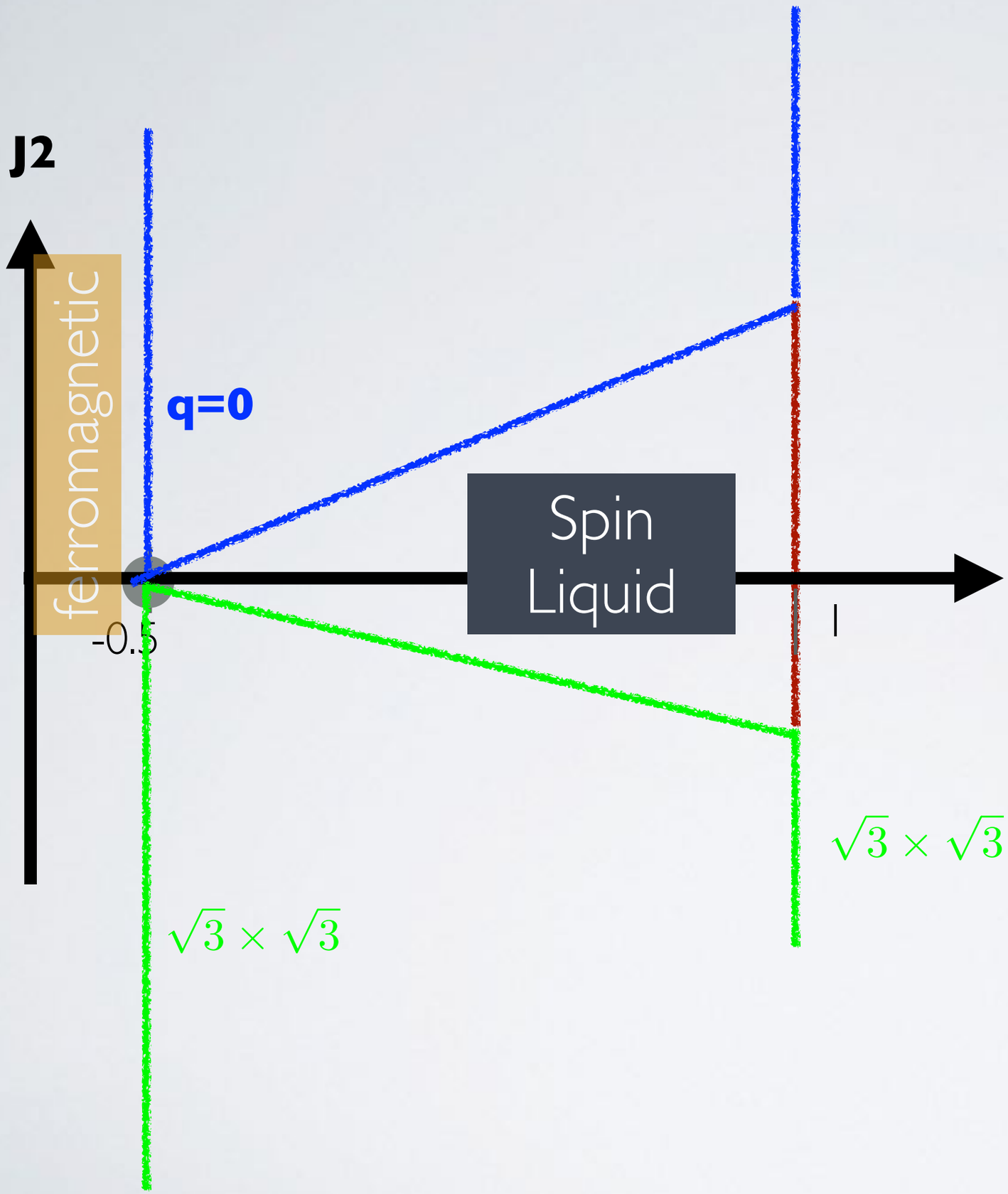

J2
ferromagnetic
q=0
Spin
Liquid
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$

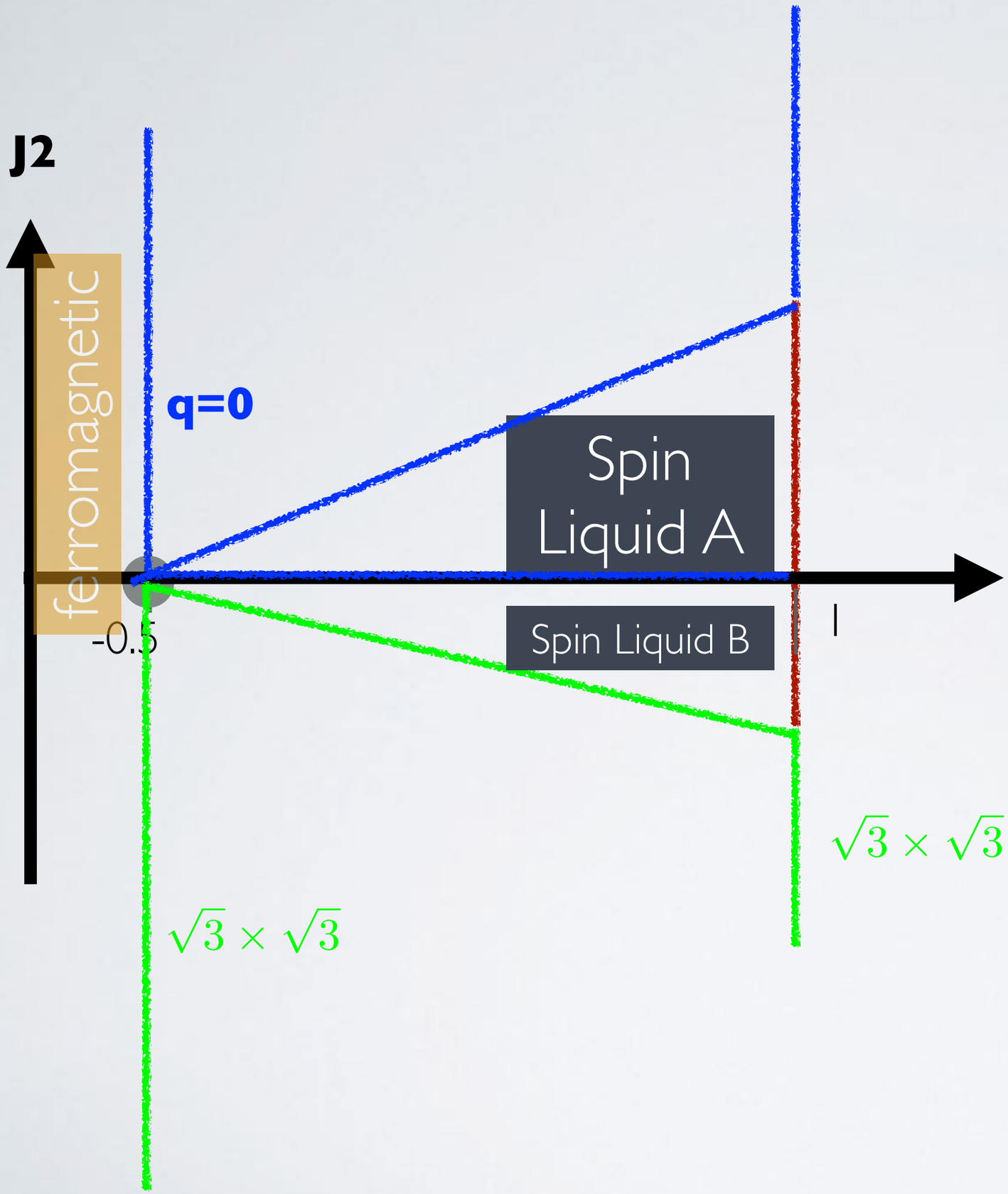
$S_z = 0$

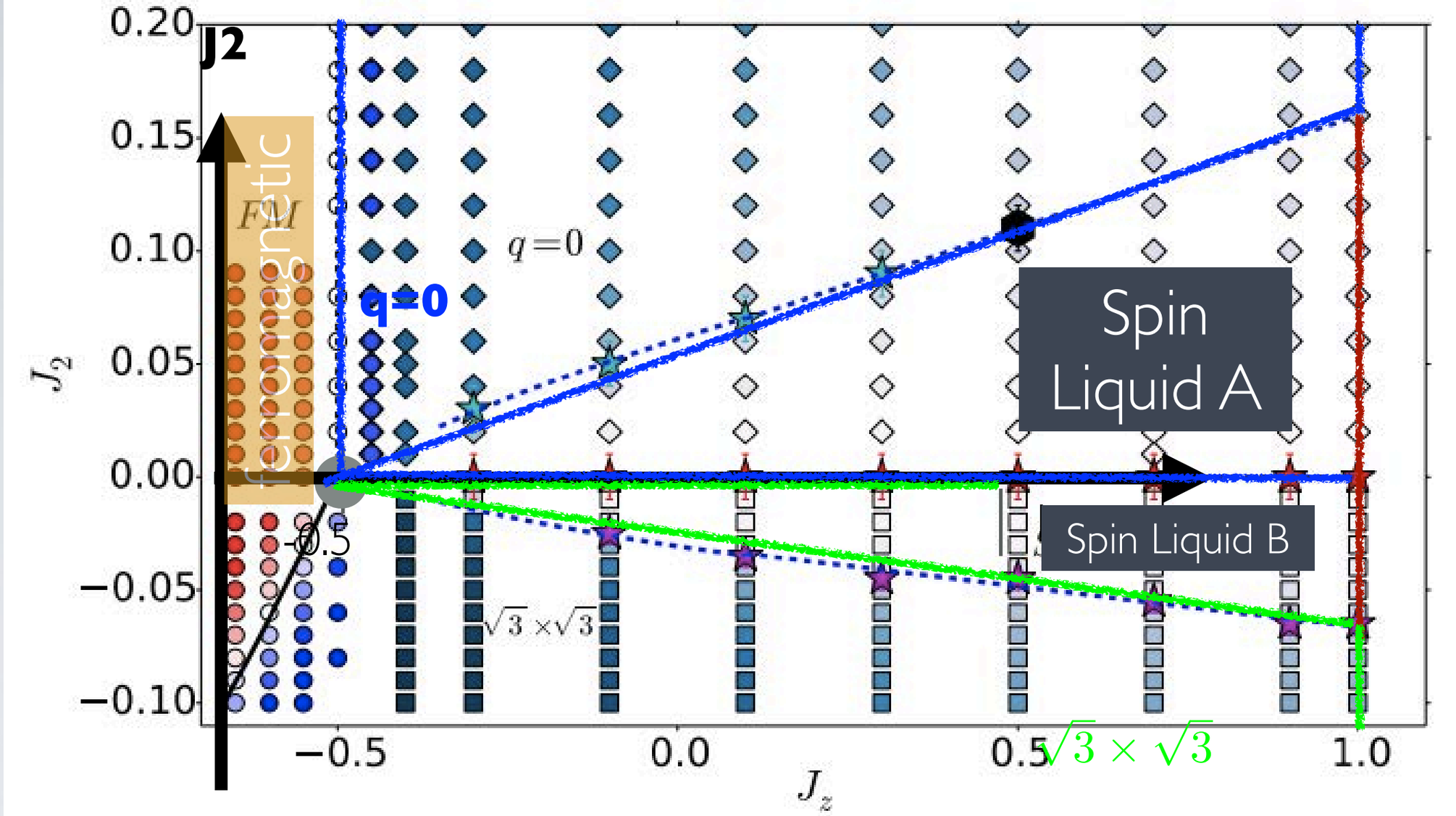

J2
ferromagnetic
q=0
Spin
Liquid A
Spin Liquid B
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$

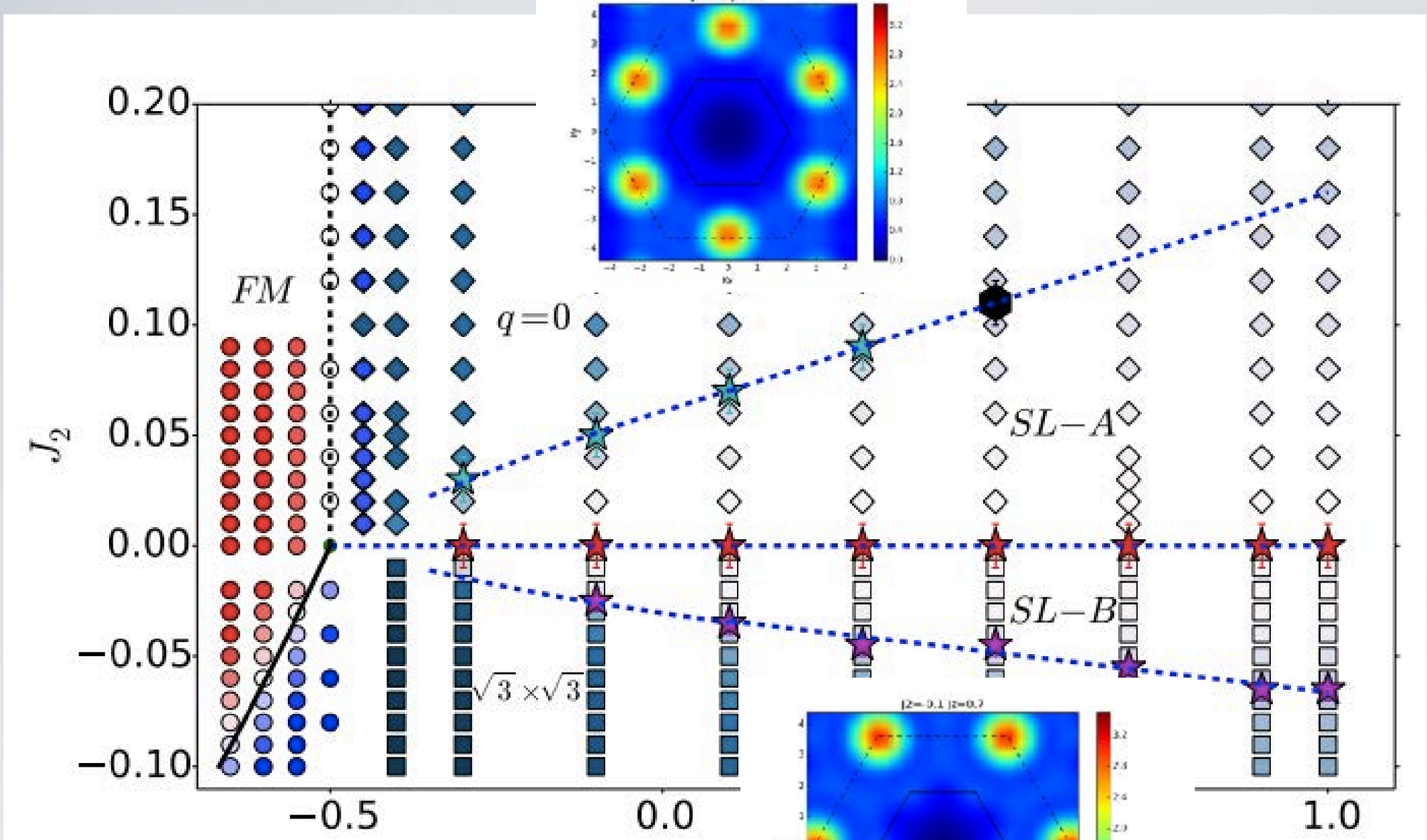
$S_z = 0$


J2
ferromagnetic
q=0
Spin
Liquid
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$

$S_z = 0$


J2
ferromagnetic
q=0
Spin
Liquid A
Spin Liquid B
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$


J2
ferromagnetic
FM
q=0
q=0
Spin Liquid A
Spin Liquid B
-0.5
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
J_2
−0.5
0.0
0.5
1.0
J_z



Q: Why co-planar states?

Colorings are all co-planar

Q: Why these co-planar states?

Fixed by colorings which satisfy J1-J2

Q: Why spin-liquids?

Q: Why so many competing phases?

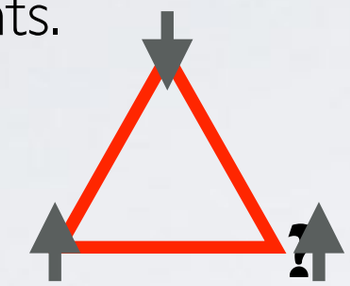
Q: Why low-energy mess?

Exponential Degeneracy

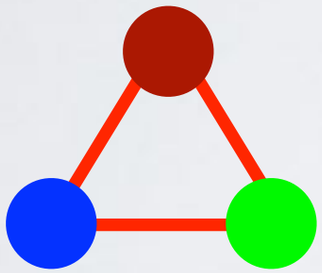
Conclusions

XXZ0 controls the physics of the Heisenberg point on lattices of pasted triangles in the way that the Ising limit doesn't.

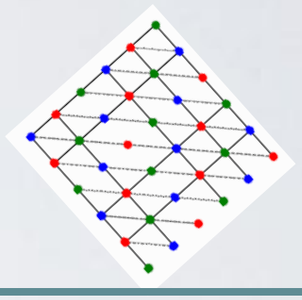
The story of frustration is not one of triangles which can't satisfy up-up-down constraints.



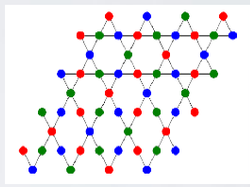
Instead, the story of frustrated magnetism is really one of coloring.



A single coloring which controls the triangular lattice.



And an exponential number of colorings which controls the kagome lattice.



From which all the known phases (and I conjecture arbitrarily many more) arise.